Shift-Invariant Image Denoising Using Mixture of Laplace Distributions in Wavelet-Domain

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Abstract. In this paper, we propose a new method for denoising of images based on the distribution of the wavelet transform. We model the discrete wavelet coefficients as mixture of Laplace distributions. Redundant, shift invariant wavelet transform is made use of in order to avoid aliasing error that occurs with critically sampled filter bank. A simple Expectation Maximization algorithm is used for estimating parameters of the mixture model of the noisy image data. The noise is considered as zero-mean additive white Gaussian. Using the mixture probability model, the noise-free wavelet coefficients are estimated using a maximum a posteriori estimator. The denoising method is applied for general category of images and results are compared with that of wavelet-domain hidden Markov tree method. The experimental results show that the proposed method gives enhanced image estimation results in the PSNR sense and better visual quality over a wide range of noise variance.

1 Introduction

Wavelets have emerged as a new mathematical tool for statistical image processing. Many image processing tasks are efficiently carried out in the wavelet-domain. Wavelets provide a compact and decorrelated image representation. The wavelet transform uses a set of basis functions, which are shifted and dilated versions of a band pass wavelet function and shifted versions of low pass scaling function. The basis functions of wavelets are localized both in time and frequency. The wavelet coefficients are computed using filter banks, where the analysis and synthesis filters form a quadrature mirror filters. For images, separable transform is constructed by applying filter bank to each column and then to each row of the result. The multiresolution nature of wavelets gives both local and global view of an image. For an image the wavelet coefficients are naturally arranged in the form of quad trees. The children coefficients in the quad trees analyze the image at one scale finer than the parent does.

The wavelet transform can be redundant. The redundancy allows enriching the set of basis functions so that the representation is more efficient in capturing information contained in an image. Many applications such as edge detection and denoising can greatly benefit from redundant representations. In noise filtering, the study the signal is required in the domain where statistics of the clean signal and the noise are modeled effectively via appropriate transforms such as the wavelet transform.

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The simplest method for wavelet-based image denoising is a thresholding rule. More advanced image denoising approaches begin with a probability model for the wavelet coefficients and then obtain an estimator via Bayesian estimation techniques, such as the MAP or MMSE estimator [1]. In this direction, wavelet-domain Hidden Markov Tree (HMT) models have demonstrated superior performance in image denoising [2]. Wavelet-domain thresholding is used to get an optimal performance in [3]. Authors in [4] have proposed bivariate shrinkage function for denoising of images by modeling non-Gaussian nature of the wavelet statistics. They have used a bivariate probability distribution function for modeling the discrete wavelet coefficients.

The key point in signal denoising is to choose appropriate probability distribution functions (pdf) that represent the wavelet coefficients and estimation of parameters of that distribution from noisy data. In this paper, we propose a nonlinear image denoising algorithm based on mixtures of Laplacian distributions for modeling the discrete wavelet coefficients.

2 Background

Multiscale image expansions implemented with filter banks offers possibility of decomposition that is shift-invariant. In image denoising applications via thresholding in the wavelet-domain, the lack of shift-invariance causes pseudo-Gibbs phenomena around singularities. To solve this problem, it is recommended to use decomposition with less shift sensitivity than the standard maximally decimated wavelet decomposition [5]. Generally, cycle spinning algorithm is employed to improve the denoising performance of a non-shift-invariant design. It is equivalent to a shift-invariant denoising if all the possible shifts of the input image are used and it is computationally more expensive.

The wavelet coefficients of natural images are generally having heavy tailed distributions and approximately uncorrelated. There exists a strong dependence on adjacent coefficients in scale and space. This suggests that multivariate Gaussian model is not accurate for wavelet-domain modeling of natural images, even though it is easy to work with such models. In wavelet-based image denoising, non-Gaussian probability models may provide superior performance in achieving high quality results.

3 Formulation of Problem

In this section, mathematical formulation of image denoising problem is explained. An image corrupted with zero-mean additive white Gaussian noise is considered. In the orthogonal wavelet domain, the problem can be formulated as $y = w + n$, where y is the noisy wavelet coefficient, w is the noise free wavelet coefficient and n is the noise. For wavelet-based denoising using distributions, it is useful to know the distribution of the clean and noisy wavelet coefficients. Let $p_w(w)$ be the probability distribution function (pdf) of w and $p_n(n)$ be the pdf of n. In [6] pdf of wavelet coefficients is modeled as a generalized Gaussian with $p_w(w) = K(s,p)\exp\left(-\left|\frac{w}{s}\right|^p\right)$, where, s, p are the parameters of the model and $K(s,p)$ is the normalization factor.