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ORIGINAL CONTRIBUTION

The use of Dimensional Analysis and Optimization of Pneumatic Drilling Operations and Operating Parameters

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Abstract Dimensional analysis was used to demonstrate the significance of these important parameters, grouped together in dimensionless numbers which will then allow for optimum use of limited laboratory data to produce better results. It allows for reduction of total effort in designing laboratory experiments, reducing total load and cost, permitting variation of the important dimensional groups rather than individual drilling operating parameters, hence a more efficient design of experiments can be realized. Drilling operations are very expensive endeavors and efforts are continuous by engineers and researchers to achieve the optimum penetration rate. To enhance bit life and penetration rate, optimization of bit design and drilling operations must be realized. To measure the penetration rate of the pneumatic drill, a fabricated pneumatic drill set up available was used. Laboratory tests were carried out to obtain the physical and mechanical properties of the rock samples. Penetration rate has been derived by means of regression statistics method. In order to overcome this drawback, dimensional analysis was used to derive relevant dimensional groups leading to the development of empirical equation of penetration rate.

Keywords Dimensional analysis · Penetration rate · Bit life · Pneumatic drill · Optimization

Introduction

Drilling is the most expensive process and the prediction of penetration rate is very important in mine planning. Total drilling cost could be estimated by using prediction equations. Also, one could use the prediction equation to select drill rig type, which is best suited for given conditions. The most significant drilling parameters are applied weight on bit, torque, rotary speed and hydraulic parameters. The most important variable in rock drilling including pneumatic drill is penetration rate. Variables used to predict penetration rate could be classified into three main categories such as, drill bit characteristics, characteristics of rock and operational variable, penetration rate of drill in rock depends on many variables. Rotational speed, thrust, bit type and diameter, blow frequency and flushing are the controllable variables. On the other hand the variables such as rock properties and geological conditions are the uncontrollable variables. Many parameters affect rock drillability, research and industry are in constant search for better models as well as experimental data. Drillability is the resistance of rock to penetration by a drilling technique operating at a standard operation. The first determinations of drillability were based on the physical properties of rocks being drilled, this proved to be ineffective.

Dimensional analysis is a mathematical tool used to resolve complex solution of the relationship between experimental variables, which are formed into dimensional groups. For predicting the penetration rate equation, dimensional analysis of Buckingham π-theorem was applied. This method allows relevant dimensional groups accounted for the penetration rate of drill in rock be established in the stage of experimental design.

This study assesses many parameters that affect on drilling performance and use of dimensional analysis to create dimensionless groups which govern the phenomenon and propose techniques for maximizing the impact of such an approach.

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Prior Art of the Work

Available literature reports several attempts to correlate drillability and various mechanical rock properties. The researchers used the modified test apparatus of Protodyakonov to determine the rock impact hardness number and develop an empirical equation for predicting drilling rates for both Down The Hole (DTH) hammer and drifter drills [1]. Protodyakonov described the Coefficient of Rock Strength (CRS) test used as a measure of the resistance of rock by impact [2]. The conventional rock properties, such as, Uniaxial Compressive Strength (UCS), Brazilian Tensile Strength (UTS), Specific Energy (SE), Shore hardness, and Mohr's hardness do not individually give good correlation with penetration rate in percussive drilling as already been discussed by earlier researchers [3]. It has been observed that UCS does not directly influence drilling rate of a drag bit [4].

The researchers developed co-relationship of rock cutting performance with intact rock properties, rock mass characteristics and cutting machine performance using dimensional analysis by grouping independent and dependent variables into dimensionless groups [5]. The researchers also developed a penetration rate model for percussive drilling using stepwise linear regression analysis [6]. The model is a function of the drill power and the physical properties of the rocks penetrated. Statistic regression was then used to analyze the relationship between the dimensionless groups. The equations developed by this method were in line with others findings.

Some investigators, used dimensional analysis for drilling performance assessment utilizing several sets of drilling data from oil fields, which could not have incorporated rock parameters that are usually determined in the lab, like petro physical parameters [7]. These researchers used 11 parameters, which, taking into account the three dimensions in the parameters, produce a total of 11 dimensionless groups.

It has also been used the dimensional analysis for relating drilling data during drilling and blasting in mining applications [8]. They used only five parameters, viz, penetration rates (R), rotational speed (N), pull down force (weight W), torque (T) and a parameter which they called 'rock quality index', (r).

Laboratory Drilling Experiments

Brief Description of the Percussive Drill Machine

The experimental set up fabricated and used in the present work was similar to the one given by earlier researchers and as shown in Fig. 1 [9]. Integral steel chisel bit of 30 mm diameter and 43 cm length was used.

Rock Samples

Pneumatic drill operation was carried out for 10 different rock samples obtained from the field. These rock samples were gabbros, basalt, soda granite, lime stone, hematite, dolerite, shale, etc. The size of the rock blocks was approximately 30 cm × 20 cm × 20 cm.

Use of Dimensional Analysis

General Information

Often, dimensional analysis is reported to when the process is very complex. This technique is used widely in solving engineering problems and scientific disciplines which enables researchers to take several parameters into account affecting a particular process or phenomenon and when detailed modeling is not available. Drilling is one such complex process and in this study dimensional analysis was used to point out the most significant parameters, using the dimensionless groups derived from the analysis.

Its application is dependent on listing of all dimensional variables affecting the process. These methods can also be helpful in correlating experimental data and

Fig. 1 Jackhammer drill setup for drilling vertical holes in rock samples

developing functional relationship between dimensional variables. Previously the researchers suggested that the derived model using dimensional analysis, which gives some of the dimensionless groups [10]. This approach, suggested in the fast for drilling applications where some dimensionless groups have been proposed with limited success.

Complex engineering problems in many fields notably fluid mechanics and heat transfer are ameanable to dimensional analysis. The technique has been successfully applied in the mechanics of solids in the study of elastic deformation and vibration of complex engineering structures [11]. It was used to establish the modeling criteria for the scale model testing of coal-face production system [12]. Its use in subsidence modeling was referred to by various scientists [13]. Backed by these literature supports the authors decided to re-apply this technique to the analysis of the drillability of rock with the intention of deriving a set of dimensionless groups, so that the results could be used to correlate the experimental data and develop appropriate functional relationships.

As mentioned above, there are many formation and drilling parameters that affect the main parameter, i.e. penetration rate. Based on published theoretical and experimental work, the rate of penetration, R (in dimensions of length over time, L/T) is dependent in a strong or a weak manner on the 12 parameters indicated in Table 1, where the dimensions are also shown, thus making a total of 12 parameters describing the process.

The Buckingham Pi Theorem

The Buckingham π (pi) Theorem states that a physical equation in general form,

$$f(X_1, X_2, X_3, \ldots, X_n) = 0 \quad (1)$$

where, the X terms are the physical quantities involved, can be reduced to one having n–m dimensionless variables where m is the number of dimensions, such that:

$$f(\pi_1, \pi_2, \pi_3, \ldots, \pi_{n-m}) = 0 \quad (2)$$

The procedure involved in reducing the original functional equation to one containing the dimensionless variables can be described as follows:

Step 1. Determine the number of π terms given by n-m

Step 2. Select the repeating variables according to the following rules.

- 2.1 The repeating variables must include among them all of the m fundamental dimensions.
- 2.2 The dependent variable should not be used as a repeating variable.

Table 1 Parameters affecting drilling process

Name	Symbol	Dimensions
Penetration rate	PR	LT^{-1}
Fluid density	ρ_f	ML^{-3}
Fluid velocity	V	MT^{-1}
Fluid viscosity	μ	$ML^{-1}\ T^{-1}$
Compressive strength of rock	σ_c	$ML^{-1}\ T^{-2}$
Tensile strength of rock	σ_t	$ML^{-1}\ T^{-2}$
Porosity of rock	Ø	–
Bulk modulus of elasticity	E	$ML^{-1}\ T^{-2}$
Bit diameter	D	L
Bit rotational speed	N	T^{-1}
Weight on bit	W	$ML\ T^{-2}$
Abrasivity of rock	A	ML^{-1}

Step 3. Assign to each value of π a different X term (not including those selected as repeating variables in Step 2).

Step 4. Find the exponents in each π term.

Step 5. Write the equations in terms of the π terms and perform such algebraic operations as may be necessary to rearrange the terms.

The Drillability of Rock

While carrying out dimensional analysis to the drillability of rock, one should commence by defining the penetration rate of a drill rock in a rock mass, in terms of those variables, which influence the process. All 12 parameters that affect the drilling process have three major units, mass, length and time (MLT), thus according to Buckingham π-theorem [14], there should be $(12 - 3) = 9$ dimensionless groups which describe the process.

Accordingly to develop the π-groups, the process chooses 3-repeating variables. The variables themselves do not form a dimensionless group, and these are, W–D–N. Following standard procedures of dimensional analysis, the nine groups can be constructed which are given in Table 2.

As a result it was found that,

$$\Pi_1 = (W)_1^a (D)_1^b (N)_1^c \cdot PR$$

$$\Pi_1 = \left(MLT^{-2}\right)_1^a (L)_1^b \left(T^{-1}\right)_1^c \cdot \left(LT^{-1}\right)$$

$$M : a_1 + 0 + 0 + 0 = 0$$

$$L : a_1 + b_1 + 0 + 1 = 0$$

$$T : -2a_1 + 0 - c_1 - 1 = 0$$

$$a_1 = 0, b_1 = -1, \ c_1 = -1$$

$$\Pi_1 = PR/DN \quad (3)$$

$$\Pi_2 = \rho_f D^4 N^2 / W \quad (4)$$

Table 2 Dimensionless groups

Name	Estimation	Coefficients	Dim. Group
Π_1	$PR^* W^a D^b N^c$	$a_1 = 0, b_1 = -1, c_1 = -1$	$\Pi_1 = PR/DN$
Π_2	$\rho f^* W^a D^b N^c$	$a_1 = -1, b_1 = 4, c_1 = 2$	$\Pi_2 = \rho_f D^4N^2/W$
Π_3	$V^* W^a D^b N^c$	$a_1 = 0, b_1 = -1, c_1 = -1$	$\Pi_3 = V/DN$
Π_4	$\mu^* W^a D^b N^c$	$a_1 = -1, b_1 = 2, c_1 = 1$	$\Pi_4 = \mu D^2N/W$
Π_5	$\sigma_c^* W^a D^b N^c$	$a_1 = -1, b_1 = 2, c_1 = 0$	$\Pi_5 = \sigma_c D^2/W$
Π_6	$\sigma_t^* W^a D^b N^c$	$a_1 = -1, b_1 = 2, c_1 = 0$	$\Pi_6 = \sigma_t D^2/W$
Π_7	Ø	–	Π_7 = Ø
Π_8	$E^* W^a D^b N^c$	$a_1 = -1, b_1 = 2, c_1 = 0$	$\Pi_8 = ED^2/W$
Π_9	$A^* W^a D^b N^c$	$a_1 = 0, b_1 = 2, c_1 = 2$	$\Pi_9 = AD^2N^2/W$

Number of variables = 9

Number of repeating variables = 3

Number of π groups = 9 – 3 = 6

$$\Pi_3 = V/DN \tag{5}$$

$$\Pi_4 = \mu D^2N/W \tag{6}$$

$$\Pi_5 = \sigma_c D^2/W \tag{7}$$

$$\Pi_6 = \sigma_t D^2/W \tag{8}$$

$$\Pi_7 = \varnothing \tag{9}$$

$$\Pi_8 = ED^2/W \tag{10}$$

$$\Pi_9 = AD^2N^2/W \tag{11}$$

$$PR/DN = \rho_f D^4N^2/W,\ V/DN, \mu D^2N/W, \sigma_c D^2/W, \sigma_t D^2/W, \varnothing,\ ED^2/W,\ AD^2N^2/W \tag{12}$$

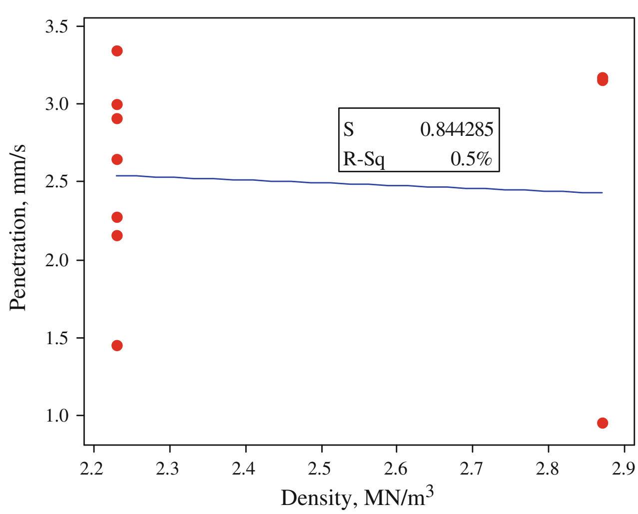
Depending on the situation, several simplifications can be made. For example, if one ignores fluid effects, i.e. when drilling with air and having sufficient air flow rate so that the bottom hole is cleaned continuously from the generated cuttings, then this can be reduced by three groups, such as fluid velocity, density and viscosity, thus the reduced equation can be,

$$PR/DN = \sigma_c D^2/W, \sigma_t D^2/W, \varnothing,\ ED^2/W,\ AD^2N^2/W \tag{13}$$

Table 3 Regression of dimensionless equation

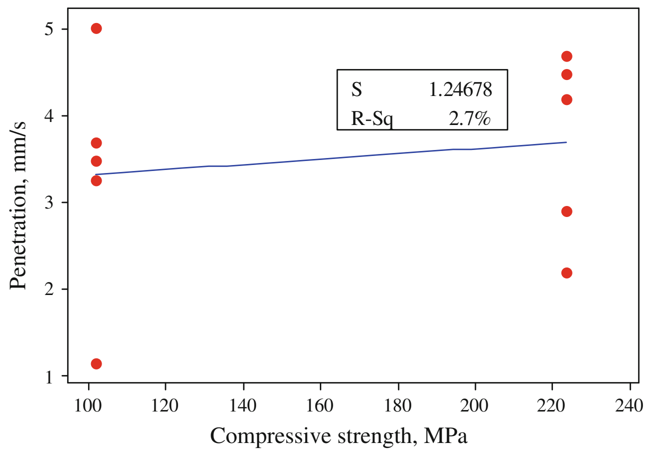
Y	X	Equation	R^2 (%)
Π_1	Π_2	Y = −0.176 X + 2.936	0.5
Π_1	Π_3	Y = 0.003063 X + 3.003	2.7
Π_1	Π_4	Y = −0.0408 X + 2.791	1.8
Π_1	Π_5	Y = −0.02209 X + 2.701	1.3

Laboratory experimentations were carried out, and the results were used to develop regression analysis of dimensionless groups of Π_1–Π_5, and these are given in Table 3 and as shown in Figs. 2, 3, 4 and 5. Considering the R^2 values, it is found that two of the most suitable variables that which are considered relevant to evaluate the predictive equation are Π_1 against Π_3 and Π_1 against Π_4. If N is assumed to be a given variable, its value is very much dependent on the feed force and rock characteristics. N is a controllable variable in many drilling rigs, which is represented by RPM. RPM can be adjusted to suit the rock condition, and the penetration rate significantly depend on N. From the statistic regression point of view, Π_3 is significantly related to the Π_4. Since the R^2 of Π_1 against Π_3 is greater than that of Π_1 against Π_4, Π_4 can no longer be included in the general equation of penetration rate. The predictive equation of the penetration rate can be



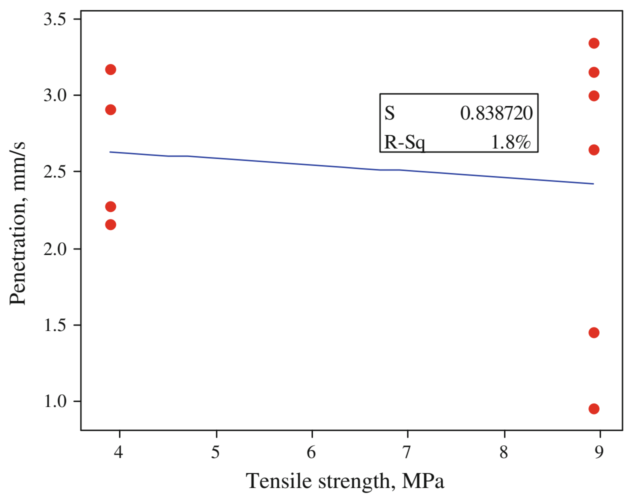
Rows	Penetration, mm/s	Density, MN/m^3
1	3.35	2.23
2	3.18	2.87
4	2.91	2.23
5	2.91	2.23
6	2.65	2.23
7	2.28	2.23
8	2.16	2.23
9	1.45	2.23
10	0.95	2.87

Fig. 2 Penetration rate against density



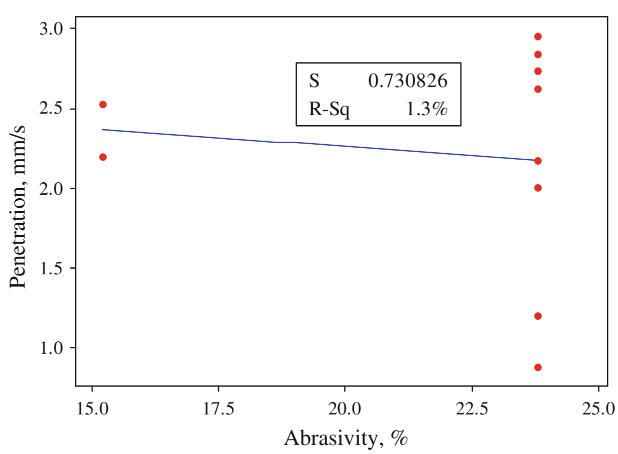
Rows	Penetration, mm/s	Compressive strength, MPa
1	5	102.05
2	4.69	223.5
3	4.48	223.5
4	4.18	223.5
5	3.69	102.05
6	3.48	102.05
7	3.26	102.05
8	2.90	223.5
9	2.19	223.5
10	1.15	102.05

Fig. 3 Penetration rate against compressive strength



Rows	Penetration, mm/s	Tensile strength, MPa
1	3.35	8.931
2	3.18	3.890
3	3.16	8.931
4	3.00	8.931
5	2.91	3.890
6	2.65	8.931
7	2.28	3.890
8	2.16	3.890
9	1.45	8.931
10	0.95	8.931

Fig. 4 Penetration rate against tensile strength



Rows	Penetration, mm/s	Abrasivity, %
1	2.95	23.8
2	2.84	23.8
3	2.74	23.8
4	2.62	23.8
5	2.53	15.2
6	2.17	23.8
7	2.20	15.2
8	2.00	23.8
9	1.2	23.8
10	0.88	23.8

Fig. 5 Penetration rate against abrasivity

developed from relationship between Π_1 against Π_3, and it can be written as follows.

$$PR = \sigma_c D^2/W, \varnothing, \ ED^2/W, \ AD^2N^2/W \quad (14)$$

Conclusion

Penetration rate of drilling using pneumatic drill for a particular range of rock types is inversely dependent with the rock strength (CS, TS) abrasivity and density. In this study penetration rate and abrasivity were taken into account. The penetration rate equation obtained from dimensional analysis is based on thrust. Penetration rate equation obtained in the experiment has a limited application, which means it can only be applied to a range of rock types that have been tested for the study. An important task for the drilling engineer is to predict the drilling rate as it allows him to choose the right drilling bit and the optimal rig operating parameters in order to optimally drill the borehole.

While dimensional analysis finds wide application in other fields like fluid mechanics, it has not been used extensively in drilling applications. The approach proposed in this study allows one to relate the main parameters affecting the drilling process. The total number of parameters affecting the drilling processes has been identified as 12. Applying the Buckingham–π theorem gives a total of 12 dimensionless groups. These analyses then allow utilizing or getting appropriate data in the proper form and use them, so that trends of the process can be identified, minimizing the amount of experimental or field work which comes from the reduction of the number of parameters to change in order to see any trends. There has not been much prior work using dimensional analysis in drilling and previous researches have utilized fewer parameters than the ones presented here.

One can see that the proposed technique allows for correct trend identification which of course is rather limited with the data set analyzed as the data lacked the monitoring of all parameters of interest.

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