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Inertia effects in circular squeeze film bearing using Herschel–Bulkley lubricants

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ABSTRACT

Recent engineering trends in lubrication emphasize that in order to analyze the performance of bearings adequately, it is necessary to take into account the combined effects of fluid inertia forces and non-Newtonian characteristics of lubricants. In the present work, the effects of fluid inertia forces in the circular squeeze film bearing lubricated with Herschel–Bulkley fluids with constant squeeze motion have been investigated. Herschel–Bulkley fluids are characterized by an yield value which leads to the formation of a rigid core in the flow region. The shape and extent of the core formation along the radial direction is determined numerically for various values of Herschel–Bulkley number and power-law index. The bearing performances such as pressure distribution and load capacity for different values of Herschel–Bulkley number, Reynolds number, power-law index have been computed. The effects of fluid inertia and non-Newtonian characteristics on the bearing performances have been discussed.

1. Introduction

Recent trends in polymer industry, thermal reactors, and in biomechanics focus immensely on the application of lubricants with variable viscosity. Further, there has been an increasing interest in the usage of non-Newtonian fluids with yield stress as lubricants, like Bingham plastic, Casson and Herschel–Bulkley fluids, as the bearing operations in machines are subjected to high speeds, loads, increasing mechanical shearing forces and continually increasing pressures.

The flow of Bingham fluid in squeeze film bearings have been analyzed and the presence and formation of yield surface have been discussed by some researchers (Covey and Stanmore [1], Gartling and Phan-Thien [2], O'Donovan and Tanner [3], Huang et al. [4]). Batra and Kandasamy [5] have theoretically analyzed the effects of fluid inertia and non-Newtonian characteristics on the bearing performances in circular squeeze film bearings using Bingham lubricants.

Adams et al. [6] have implemented a finite element analysis of squeeze flow for a material that exhibits elasto-viscoplasticity and the numerical simulations are verified with the experimental measurements. Based on Adams et al. [6], squeeze flow experiments have been carried out by Sherwood and Durban [7] for a Herschel–Bulkley fluid using lubricated wall boundary conditions. A simple expression for the total force required to push the plates together have also been verified for fluid at high strain in the same work. Chan and Baird [8] have analyzed the deviations of the simulated finite element results obtained using the lubrication approximation, with that of experimental results for the squeezing flow of a Herschel–Bulkley fluid. Meeten [9] has carried both theoretical and experimental investigation of the force required to squeeze a Herschel–Bulkley material without slip for different surface patterns. Zhang et al. [10] have numerically analyzed the behavior of Herschel–Bulkley fluids in between two ellipsoid rollers. The problem of circular squeeze film bearing using

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Herschel–Bulkley fluids as lubricants has been studied by Kandasamy et al. [11]. This study considered the equation of motion under the lubrication approximations in the inertia-less frame work. The focus of that study was mainly on the thickness of the "rigid" core formed in the region between the plates. The results of load capacity for various yield numbers and flow behavior indices have also been computed in the inertia-less regime.

In the present work, the problem of a circular squeeze film bearing using Herschel–Bulkley fluid as a lubricant with inertia effects has been analyzed. During the operations of the bearings, the maximum viscous shearing stresses arise in the region between the plates. Therefore, there may be a region in the film where the shearing stresses do not exceed the yield value of the lubricant and thereby a core with zero velocity gradient is formed. The flow occurs only in the region where the shear stress exceeds the yield value. The shape and extent of the core along the radius for various values of the Herschel–Bulkley number and power-law index for the case of constant squeeze motion has been numerically determined. The flow is confined to the region between the core and the circular plates of the bearing. Numerical solutions have been obtained for the bearing performances such as pressure distribution and load capacity for different values of Herschel–Bulkley number, power-law index and Reynolds number.

2. Mathematical Formulation of the problem

The geometry of the problem is as shown in Fig. 1. We consider an isothermal, incompressible, steady flow of a time independent Herschel–Bulkley fluid squeezed between two circular plates separated by a distance h. Let $2R$ be the diameter of the bearing approaching each other with a squeeze velocity v_s under a normal load W. We consider cylindrical polar co-ordinates (r, θ, z) with axial symmetry at the center of the plate. Let v_r and v_z represent velocity components in the radial and axial directions, respectively, and ρ denote the density of the fluid. It is assumed that there is no sliding motion of the two plates.

The constitutive three-dimensional equation of Herschel–Bulkley fluids is given by (Alexandrou et al. [12]),

$$\tau = \left\{ \eta_1 \left(\frac{D_{II}}{2}\right)^{\frac{(n-1)}{2}} + \frac{\eta_2\left[1-\exp\left(-m\left|\sqrt{D_{II}/2}\right|\right)\right]}{\sqrt{D_{II}/2}} \right\} D, \tag{1}$$

where τ are the deviatoric stress components, η_2, η_1, m and n are constants named the yield stress, consistency index, stress exponent growth and power-law index, respectively. Here $D = [\nabla u + \nabla u^T]$ represents the rate of deformation components and $D_{II} = D_{ij}D_{ij}$ is second strain invariant. Practical examples of such materials are greases, colloidal suspensions, starch pastes, silicon suspensions and such other fluids. Further, for all practical purposes we can use the one-dimensional analog of Eq. (1) given by,

$$\tau = \eta_2 + \eta_1 \dot{\gamma}^n, \tag{2}$$

where $\dot{\gamma}$ represents the shear rate.

In those regions of the film, where the shear stress is less than the yield value, there will be a core formation, which will move with constant velocity v_c. Let the boundaries of the core be given by $z = h_1(r)$ and $z = h_2(r)$ as shown in Fig. 2.

Applying the basic assumptions of lubrication theory for thin films, the governing equations for the above squeeze film system, including inertia forces, is given by,

$$\rho\left(v_r\frac{\partial v_r}{\partial r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r} + \frac{\partial \tau_{rz}}{\partial z}, \tag{3}$$

$$\frac{\partial p}{\partial z} = 0, \tag{4}$$

$$\frac{1}{r}\frac{\partial (r v_r)}{\partial r} + \frac{\partial v_z}{\partial z} = 0, \tag{5}$$

$$\tau_{rz} = \eta_2 + \eta_1\left(\frac{\partial v_r}{\partial z}\right)^n. \tag{6}$$

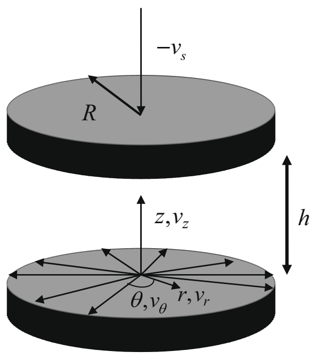

Fig. 1. Geometry of the squeeze film bearing.

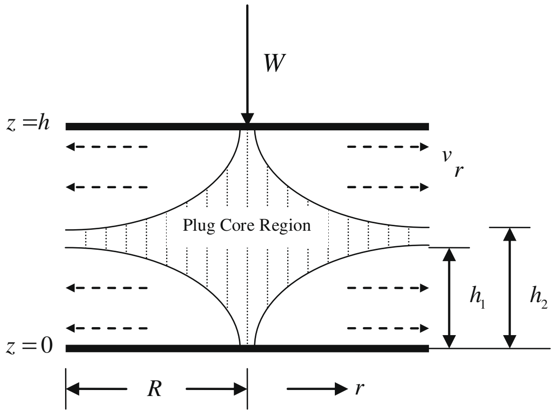

Fig. 2. Shape of the core.

The above Eqs. (3), (4) and (6) together with the continuity Eq. (5), are to be solved under the following boundary conditions:

$$\begin{aligned} &v_r = 0 \quad \text{at} \quad z = 0, h, \\ &v_z = 0 \quad \text{at} \quad z = 0, \\ &v_z = -v_s \quad \text{at} \quad z = 0, \\ &p = p_a \quad \text{at} \quad r = R, \end{aligned} \tag{7}$$

v_r and $\frac{\partial v_r}{\partial z}$ are continuous at $z = h_1(r)$ and $z = h_2(r)$. Here p_a is the atmospheric pressure.

3. Solution to the problem

The integral form of the continuity equation, also called the equation of squeeze motion is given by,

$$2\pi r \int_0^h v_r dz = \pi r^2 v_s. \tag{8}$$

Averaging the inertia terms in the momentum Eq. (3) by assuming it to be a constant over the film thickness (Hashimoto and Wada [13]), and performing integration by parts in the resulting equation using the continuity Eq. (5) and boundary conditions, we get,

$$\frac{\rho}{h}\left[\frac{\partial}{\partial r}\int_0^h v_r^2 dz + \frac{1}{r}\int_0^h v_r^2 dz\right] + \frac{dp}{dr} = \frac{\partial \tau_{rz}}{\partial z}. \tag{9}$$

Here, we introduce the following modified pressure gradient:

$$f \equiv \frac{\rho}{h}\left[\frac{\partial}{\partial r}\int_0^h v_r^2 dz + \frac{1}{r}\int_0^h v_r^2 dz\right] + \frac{dp}{dr}. \tag{10}$$

Hence, from Eqs. (9) and (10), we have,

$$\frac{\partial \tau_{rz}}{\partial z} = f. \tag{11}$$

As the pressure gradient is independent of the co-ordinate z, Eq. (11) can be integrated as follows:

$$\tau_{rz} = fz + c_1. \tag{12}$$

Substituting τ_{rz} from Eq. (12) into Eq. (6) and integrating the resulting equations using the boundary conditions (7), we get the velocity distribution in the two flow regions separating the core region as,

$$v_r = \left(\frac{f}{\eta_1}\right)^{\frac{1}{n}}\left(\frac{n}{n+1}\right)\left[(z-h_1)^{\frac{1}{n}+1} - (-h_1)^{\frac{1}{n}+1}\right], \tag{13}$$

in $0 \leqslant z \leqslant h_1(r)$,

$$v_r = \left(\frac{f}{\eta_1}\right)^{\frac{1}{n}}\left(\frac{n}{n+1}\right)\left[(z-h_2)^{\frac{1}{n}+1} - (h-h_2)^{\frac{1}{n}+1}\right], \tag{14}$$

in $h_2(r) \leqslant z \leqslant h$ and the core velocity as,

$$v_c = -\left(\frac{f}{\eta_1}\right)^{\frac{1}{n}}\left(\frac{n}{n+1}\right)(-h_1)^{\frac{1}{n}+1} = \left(\frac{f}{\eta_1}\right)^{\frac{1}{n}}\left(\frac{n}{n+1}\right)(h-h_2)^{\frac{1}{n}+1}, \tag{15}$$

in $h_1(r) \leqslant z \leqslant h_2(r)$. From Eq. (15), we have,

$$h_1(r) = h - h_2(r). \tag{16}$$

Considering the equilibrium of an element of the core in the fluid, we get,

$$f = -\frac{2\eta_2}{H}, \tag{17}$$

where $H = H(r) = h_2(r) - h_1(r)$, (18)

represents the thickness of the core.

Using the velocity Eqs. (13)–(15) in the equation of squeeze motion (8), we get,

$$f = -\frac{2\eta_1[(n+1)(2n+1)rv_s]^n}{(h-H)^{n+1}[n\{nH+(n+1)h\}]^n}. \tag{19}$$

Eliminating f from Eqs. (17) and (19), we get an algebraic equation for determining the thickness of the core as,

$$\frac{n(h-H)^{\frac{1}{n}+2}}{(2n+1)} + \left(\frac{n+1}{n}\right)\left(\frac{r^n v_s^n \eta_1 H}{\eta_2}\right)^{\frac{1}{n}} - h(h-H)^{\frac{1}{n}+1} = 0. \tag{20}$$

Further, substituting the velocity Eqs. (13)–(15) in Eq. (9), we get,

$$\frac{dp}{dr} = \left[\frac{-2\eta_1}{(h-H)^{n+1}}\right]\left[\frac{(n+1)(2n+1)rv_s}{n\{nH+(n+1)h\}}\right]^n - \left(\frac{\rho v_s^2 r}{4h}\right)[K_1 - K_2], \tag{21}$$

where

$$K_1 = \left[\frac{(2n+1)\left[3\{2(n+1)^3h^2 + (4n+3)n^2H^2 + n(6n^2+11n+5)hH\}\right]}{(3n+2)(h+nh+nH)^3}\right], \tag{22}$$

$$K_2 = \left[\frac{(2n+1)\left[n\{(n+1)h + (4n+3)nH\}r\frac{dH}{dr}\right]}{(3n+2)(h+nh+nH)^3}\right]. \tag{23}$$

Now, the following non-dimensional quantities are introduced:

$$H^* = \frac{H}{h}, \quad r^* = \frac{r}{R}, \quad p^* = \frac{h^{2n+1}p}{v_s^n R^{n+1}\eta_1}, \quad p_a^* = \frac{h^{2n+1}p_a}{v_s^n R^{n+1}\eta_1}. \tag{24}$$

Then, the non-dimensional form of Eq. (20) can be expressed as,

$$\frac{n(1-H^*)^{\frac{1}{n}+2}}{(2n+1)} + \left(\frac{n+1}{n}\right)r^*\left(\frac{H^*}{N}\right)^{\frac{1}{n}} - (1-H^*)^{\frac{1}{n}+1} = 0, \tag{25}$$

where $N = \frac{\eta_2 h^{2n}}{\eta_1 R^n v_s^n}$, is called the yield number or Herschel–Bulkley number.

The core thickness can be determined numerically from the above nonlinear algebraic equation using any iterative technique. The root of Eq. (25), $H^*(r^*, n, N)$ which is positive and less than unity determines the yield surface in the region between the plates. The value of H^* is determined for various values of r^*, n, N.

Further, the expression for pressure distribution can be obtained by non-dimensionalising and integrating Eq. (21) together with the boundary conditions as given below:

$$p^* - p_a^* = \int_1^{r^*}\left[-\left[\frac{2^{\frac{1}{n}}(n+1)(2n+1)r^*}{n\{nH^* + (n+1)\}(1-H^*)^{1+\frac{1}{n}}}\right]^n - Re[K_1 - K_2]r^*\right]dr^*, \tag{26}$$

where

$$K_1^* = \left[\frac{(2n+1)[3\{2(n+1)^3 + (4n+3)n^2H^{*2} + n(6n^2+11n+5)H^*\}]}{4(3n+2)(1+n+nH^*)^3}\right], \tag{27}$$

$$K_2^* = \left[\frac{(2n+1)\left[n\{(n+1) + (4n+3)nH^*\}r^*\frac{dH^*}{dr^*}\right]}{4(3n+2)(1+n+nH^*)^3}\right], \tag{28}$$

and $Re = \frac{\rho h^{2n-1}}{v_s^{n-2} R^{n-1} \eta_1}$, is the Reynolds number. The pressure distribution in the squeeze film bearing can be obtained for different values of yield number, power-law index and Reynolds number numerically.

The load capacity of the circular squeeze film bearing is then given by,

$$W = \int_0^1 (p^* - p_a^*) r^* dr^*. \tag{29}$$

This integral is evaluated numerically and the load values are obtained for various values of power-law index, squeeze reynolds number and yield number.

4. Results and discussion

The thickness of plug core formed in the region between circular plates due to the pressure gradient for various values of power-law index n and Herschel–Bulkley number N along the radius r^* are shown in the Figs. 3–5. A similar shape was observed by Covey and Stanmore [1] and by Gartling and Phan-Thien [2] through numerical simulation. The core thickness is maximum at the center and minimum at the periphery. It can be observed that core thickness increases with increase in the Herschel–Bulkley number for a particular value of power-law index. Further, the core thickness decreases with increase in power-law index for a particular Herschel–Bulkley fluid.

The distribution of pressure for various values of the Reynolds number Re, Herschel–Bulkley number and power-law index in the radial direction are as shown in the Figs. 6–8. The pressure is found to be maximum at the center and decreases along the radial direction. However, there is a significant increase in the pressure for the fluids with high Herschel–Bulkley number at a particular power-law index, but the increase in pressure due to increase in Reynolds number is not significant. Further, there is a considerable increase in the pressure distribution due to the increase in power-law index of the fluid.

The results of load carrying capacity for various values of power-law index, Herschel–Bulkley numbers and Reynolds numbers are tabulated in the Tables 1–3. From these tables, we observe that the load capacity of the bearing increases with the increase of the power-law index for the fluids with a particular Herschel–Bulkley number. But the percentage increase in the load capacity decreases with increase of Herschel–Bulkley number. Further, the load capacity of the bearing increases significantly with the increase in Herschel–Bulkley number for a particular value of power-law index.

Fig. 9 depicts the percentage increase in load capacity due to the increase in Reynolds number for various values of power-law index and Herschel–Bulkley number. It is observed that load capacity variation between pseudoplastic yield stress fluids ($n < 1.0$) and Dilatant yield stress fluids ($n > 1.0$) at a particular Reynolds number is insignificant. Further, percentage increase in load capacity due to inertia effect is observed to be significant only for fluids with low Herschel–Bulkley numbers but for fluids with large Herschel–Bulkley numbers, the effects of inertia are negligible.

The plots corresponding to $n = 1.0$ in all the above graphs depicts the behavior of bearing performances when Bingham fluid is used as a lubricant and the results are in agreement with the results obtained by Batra and Kandasamy [5]. The analytical solution for load carrying capacity using Newtonian fluid as a lubricant is $\frac{3}{4} + \frac{9}{80} Re$, where Re is the Reynolds number. Newtonian fluid can be obtained as limiting case of Herschel–Bulkley fluid when $N = 0$ and $n = 1.0$. Fig. 10 depicts that as $n = 1.0$ and N tends to zero the results match with the analytical solution of the Newtonian fluid. Further, the results corresponding to $Re = 0$ which exhibit the inertia-less effects on the pressure and load capacity of the bearing match with the results of Kandasamy et al. [11].

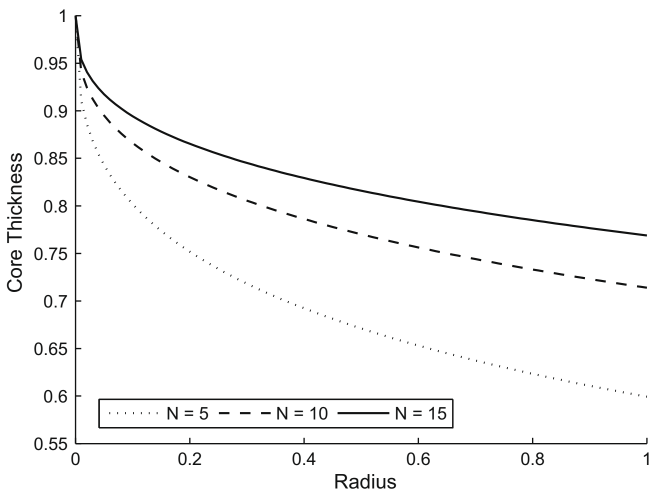

Fig. 3. Core thickness variation along the radius for $n = 0.6$.

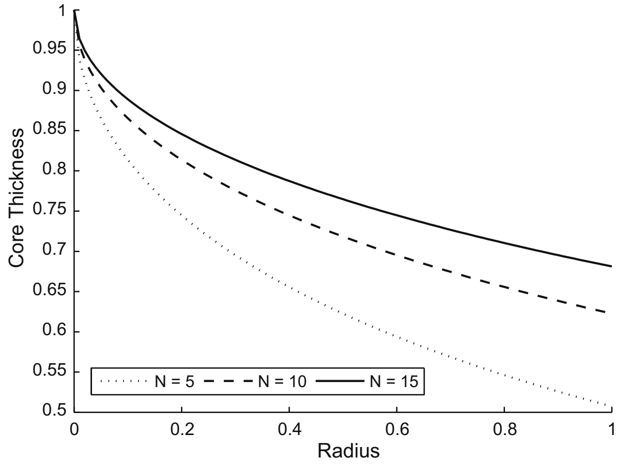

Fig. 4. Core thickness variation along the radius for $n = 1.0$.

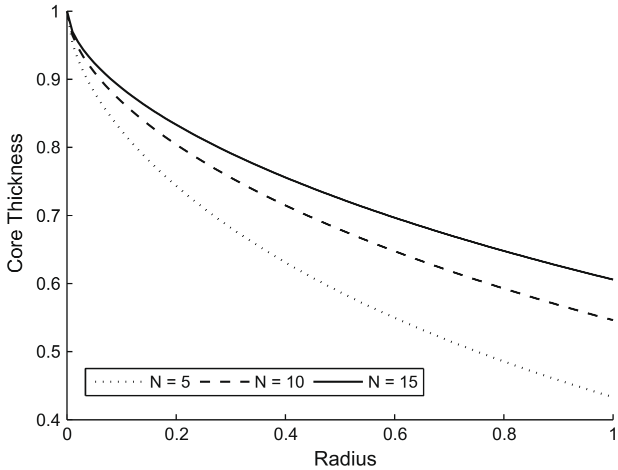

Fig. 5. Core thickness variation along the radius for $n = 1.4$.

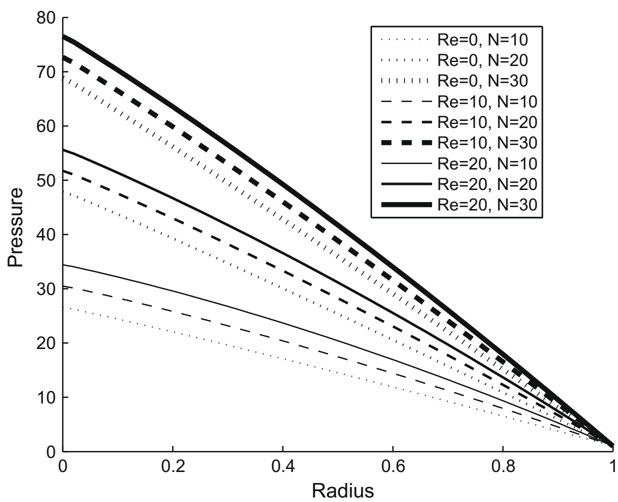

Fig. 6. Pressure distribution along the radius for $n = 0.6$.

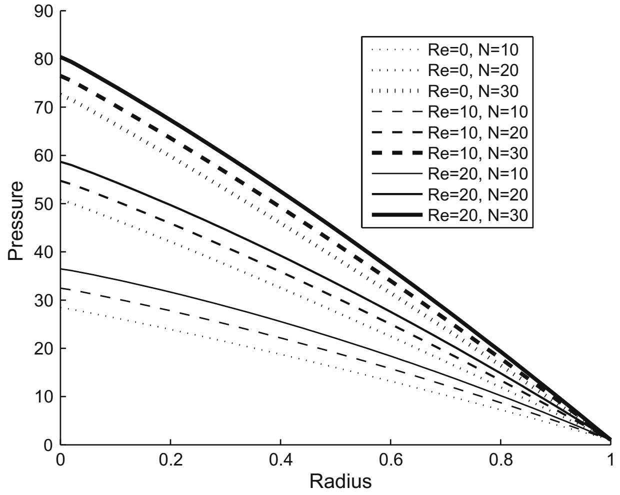

Fig. 7. Pressure distribution along the radius for $n = 1.0$.

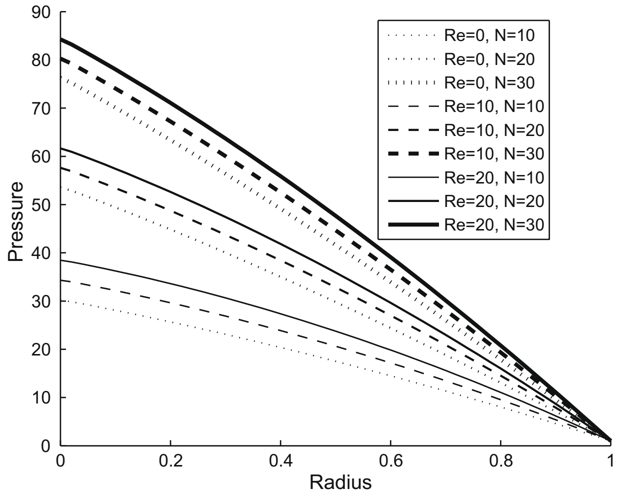

Fig. 8. Pressure distribution along the radius for $n = 1.4$.

Table 1
Load capacity values for N and n for $Re = 0$.

n	N		
	1	2	3
0.6	1.0521	1.4665	1.8667
1.0	1.2432	1.7020	2.1427
1.4	1.4497	1.9478	2.4258

Table 2
Load capacity values for N and n for $Re = 10$.

n	N		
	1	2	3
0.6	2.0994	2.4937	2.8814
1.0	2.3339	2.7713	3.1976
1.4	2.5719	3.0500	3.5135

Table 3
Load capacity values for N and n for $Re = 20$.

n	N		
	1	2	3
0.6	3.1466	3.5209	3.8960
1.0	3.4247	3.8406	4.2525
1.4	3.6941	4.1521	4.6013

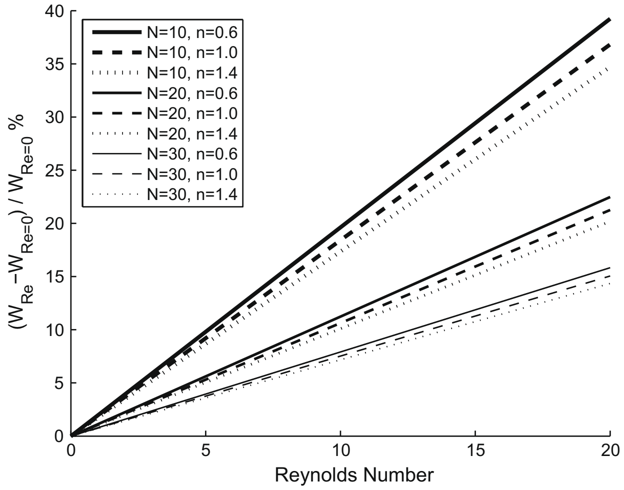

Fig. 9. Percentage increase in the inertia effect of the load capacity.

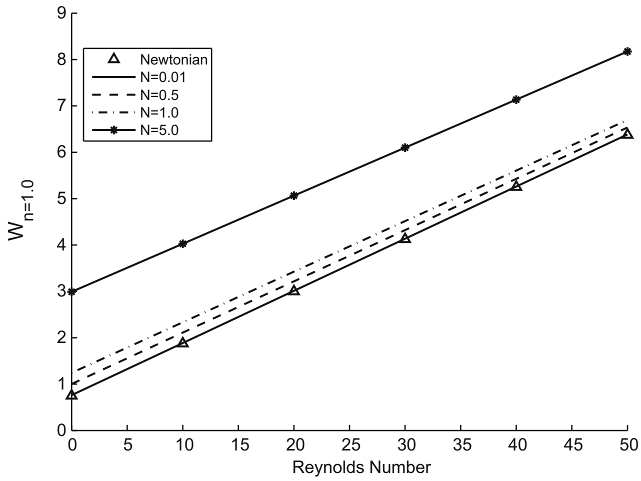

Fig. 10. Validation of the load carrying capacity of the squeeze film bearing.

5. Conclusions

The shape and extent of the core formation along the radial direction is determined numerically for various values of Herschel–Bulkley number and power-law index. It is found that the core thickness is high for the fluids with high Herschel–Bulkley number. The bearing performances such as pressure distribution and load capacity for different values of Herschel–Bulkley number, Reynolds number, power-law index have been computed. It is observed that the pressure and thereby the load capacity of the bearing are proportionately increasing with the increase of Herschel–Bulkley number and power-law index of the fluids. But the percentage increase due to inertia is not very significant for the fluids with high Herschel–Bulkley numbers.

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