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Enhanced PID Controller for Non-Minimum Phase Second Order Plus Time Delay System

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Abstract:
A tuning method is developed for the stabilization of the non-minimum phase second order plus time delay systems. It is well known that the presence of positive zeros pose fundamental limitations on the achievable control performance. In the present method, the coefficients of corresponding powers of s, s^2 and s^3 in the numerator are equated to α, β and γ times those of the denominator of the closed-loop system. The method gives three simple linear equations to get the PID parameter. The optimal tuning parameters α, β and γ are estimated by minimizing the Integral Time weighted Absolute Error (ITAE) for servo problem using *fminsearch* MATLAB solver aimed at providing lower maximum sensitivity function and keeping in check with the stability. The performance under model uncertainty is also analysed considering perturbation in one model parameter at a time using Kharitonov's theorem. The closed loop performance of the proposed method is compared with the methods reported in the literature. It is observed that the proposed method successfully stabilizes and improves the performance of the uncertain system under consideration. The simulation results of three case studies show that the proposed method provides enhanced performance for the set-point tracking and disturbance rejection with improved time domain specifications.

Keywords: PID controller, non-minimum phase system, IMC method, Kharitonov's theorem

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1 Introduction

A system is said to be non-minimum phase (NMP) whose transfer function has one of the zeros lying on the right half of S-plane; which leads the step response in the direction opposite to that of the initial steady state path. This response is also known as an inverse response in time domain. The modulus of the non-minimum phase response is greater than that of minimum phase system with the same amplitude response. The phase angle of any non-minimum phase system is greater than *90* degrees. It is well known that the presence of zeros pose fundamental limitations on the achievable control performance. Systems with positive zeros are fundamentally and quantifiably more difficult to control than the systems without zero [1]. The closed loop response of a SOPTD system with a zero shows a larger overshoot compared to that of the system without zero. The closed loop performance of the control system is complicated by the presence of a zero.

The method of designing the controller for higher order system is limited in open literature. Therefore, the higher order models are approximated to First order plus time delay (FOPTD) in most of the cases. However, the approximate FOPTD model provides a negative time constant on few occasions, in such cases the higher order models can be approximated to second order plus time delay (SOPTD) instead of FOPTD [2]. The SOPTD model can capture the dynamics of the higher systems better than the FOPTD models. Furthermore, the controller settings designed using the SOPTD model provide better closed loop performance than the controller designed by the FOPTD model [3]. There are many systems, which exhibit the second order plus time delay with positive zero system like that of the drum boiler [4]. The non-minimum phase systems are sluggish in response because of their under shoot at the start of the response. Presence of the positive zero complicates the performance of the dynamics of the control system and limits the maximum bandwidth. Internal stability is one of the main problems with the NMP system.

Proportional Integral Derivative (PID) controllers are widely used in the automation industry due to their relative simplicity and the satisfactory performance they provide for a wide range of processes [5]. PID controllers continue to be a very active research field, as they are often poorly tuned in industries and the performances obtained can be improved many times by applying a more efficient PID control technology. Although various control techniques for non - minimum phase systems have been developed, it is no surprise that for a

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few years and even now, the classical PID controllers have been widely used in industry and research applications. In addition, PID controllers have many important features such as feedback, the ability to remove steady state offset by integral action, and the future can be anticipated by derivative action [6]. A Simplified Filtered Smith Predictor has been designed for stable, integrative, and unstable FOPDT models. They demonstrated that the order and complexity of the Filtered Smith Predictor filter is not necessary to increase in order to deal with the noise if the primary control unit is properly selected to adjust the set point response [7]. A dead time compensator algorithm has been reported in literature for dealing the multiple delay SISO systems. Two FIR filters are used in the algorithm to obtain a required tracking point [8]. Torrico et al. [9] introduce a novel tuning technique, for which robust loops and noise attenuation can be used simultaneously for stable, unstable and integrating dead time processes. Compared to more complex controllers, the lower order filters are shown to meet design requirements and provide better performance.

Methods for the design of PI/PID controllers for stable non-minimum phase systems have been reported in open literature such as IMC method [10, 11], phase margin and margin method [12], optimization method [13, 14], etc. The equating coefficient method has been derived for the integrator dead time process [15] and extended to an unstable First Order plus Time Delay (FOPTD) process [10] and the inverse response systems. The modified IMC have been developed for the inverse response system [16]. Later on, an IMC and stability analysis method was presented for the unstable SOPTD model [17]. IMC method has been used for first order unstable/stable system with zero [18]. To overcome the inherent problem of internal stability, a modified internal model control technique has been developed to track the set point and load disturbance rejection [19]. An open loop unstable system has a time delay limitation on system performance that has been addressed by implementing two degree of freedom control structure [20]. A time delay compensators with different structures has been given for stable, integrating and unstable processes [21]. For the USOPTD non-minimum phase system, a modified form of the Smith predictor method has been tested to improve the set point and load rejection response [22]. Using simulation studies, it is shown that an enhanced IMC-based PID controller can be used to control non-minimum phase integrating systems effectively [23].

There are several methods available in literature, which can be used to design PI/PID controller for non-minimum phase FOPTD systems. However, limited methods are available for non-minimum phase SOPTD process. In this work, we estimate a PID settings for NMP SOPTD. The design procedure of all the above-mentioned methods is rather complicated. In this work, we present a simple method to find a PID parameter that solves three simple linear equations. In this work, we present a simple method, which solve three linear equations to find PID parameter. This method equals the degree of a numerator and denominator of the closed loop transfer function of servo problem and gets the equations on equating the powers of s, s^2, and s^3 of the numerator and the denominator. It is, therefore, possible to obtain the PID parameter by solving these equations. In order to know the effectiveness of the proposed method, simulation studies are carried out, and the performance is compared with the reported work.

2 Proposed method

The general form of Second Order Plus Time Delay with Zeros (SOPTDZ) system is given as:

$$G = \frac{k_p(1-ps)e^{-Ls}}{as^2 + bs + 1} \tag{1}$$

PID controller is considered here. The PID controller equation can be written by the following equation

$$G_c = \frac{u(s)}{e(s)} = k_c[1 + \frac{1}{\tau_I s} + \tau_D s] \tag{2}$$

where u is manipulated variable and e is error (i. e. $y - y_r$)

The closed loop transfer function relating the output variable (y) and set point (y_r) can be given by the following expression [24]

$$\frac{y}{y_r} = \frac{GG_C}{1 + GG_C} \tag{3}$$

Substituting G and G_c in the above equation and rearranged to get the following equation

$$\frac{y(s)}{yr(s)} = (k_1 s + k_2 + k_3 s^2) \times \frac{(1-ps)e^{(-Ls)}}{s(as^2 + bs + 1) + (1-ps)(k_1 s + k_2 + k_3 s^2)e^{-Ls}} \tag{4}$$

Where,

$$k_1 = k_c k_p \tag{5}$$

$$k_2 = \frac{k_1}{\tau_I} \tag{6}$$

$$k_3 = k_1 \tau_D \tag{7}$$

The exponential term (e^{-Ls}) in the numerator is eliminated for further analysis, as it only shifts the corresponding time axis. The exponential term (e^{-Ls}) in the denominator can be written as

$$e^{-Ls} = \frac{e^{-0.5Ls}}{e^{0.5Ls}} \tag{8}$$

$$\frac{y(s)}{yr(s)} = (k_1 s + k_2 + k_3 s^2) \times \frac{(1-ps)e^{(0.5Ls)}}{s(as^2 + bs + 1)e^{0.5Ls} + (1-ps)(k_1 s + k_2 + k_3 s^2)e^{-0.5Ls}} \tag{9}$$

Eq. (9) can be rewritten as

$$\frac{y(s)}{yr(s)} = \frac{(k_1 s + k_2 + k_3 s^2)(1-ps)e^{0.5Ls}}{s(as^2 + bs + 1)e^{0.5Ls} + (1-ps)(k_1 s + k_2 + k_3 s^2)e^{-0.5Ls}} \tag{10}$$

The following expression can be obtained after expanding the exponential terms $e^{(0.5Ls)}$ and $e^{(-0.5Ls)}$ using the Taylor series and substituting them in the above equation.

$$\frac{y(s)}{yr(s)} = \frac{(a_3 s^3 + a_2 s^2 + a_1 + k_2)}{(b_3 s^3 + b_2 s^2 + b_1 s + k_2)} \tag{11}$$

Where,

$$a_3 = -pk_3 + 0.5k_3 L - 0.5pk_1 L + 0.125k_1 L^2 - 0.125pk_2 L^2 + 0.0208k_2 L^3 \tag{12}$$

$$b_3 = -pk_3 - 0.5k_3 L + 0.5pk_1 L + 0.125k_1 L^2 - 0.125pk_2 L^2 - 0.0208k_2 L^3 + a + 0.5bL + 0.125L^2 \tag{13}$$

$$a_2 = k_3 - pk_1 + 0.5k_1 L - 0.5pk_2 L + 0.125pk_2 L^2 \tag{14}$$

$$b_2 = k_3 - pk_1 - 0.5k_1 L + 0.5pk_2 L + 0.125pk_2 L^2 + b + 0.5L \tag{15}$$

$$a_1 = k_1 - pk_2 + 0.5k_2 L \tag{16}$$

$$b_1 = k_1 - pk_2 - 0.5k_2 L + 1 \tag{17}$$

Numerator and denominator of eq. (11) are taken into account. The order of the numerator and denominator is the same, and the presence of the integral action equals the constant term in both the numerator and denominator. Considering open loop stable systems and equating the coefficient of the corresponding powers of s of numerator and denominator, with the controllers objective to make $y/y_r = 1$. On equating the powers of s^3 of numerator α times that of the denominator of eq. (11) i. e.$a_3 = \alpha b_3$

We get,

$$- pk_3(1 - \alpha) + 0.5k_3L(1 + \alpha) - 0.5pk_1L(1 + \alpha) + 0.125k_1L^2(1 - \alpha) - 0.125pk_2L^2(1 - \alpha) + 0.0208k_2L^3(1 + \alpha) - (a + 0.5bL + 0.125L^2)\alpha = 0 \tag{18}$$

On equating the powers of s^2 of numerator β times the denominator of eq. (11) i. e.$a_2 = \beta b_2$ we get,

$$k_3(1 - \beta) - pk_1(1 - \beta) + 0.5k_1L(1 + \beta) - 0.5pk_2L(1 + \beta) + 0.125pk_2L^2(1 - \beta) - \beta(b + 0.5L) = 0 \tag{19}$$

On equating the powers of s of numerator γ times the denominator of eq. (11) i. e. $a_1 = \gamma b_1$ we get,

$$k_1(1 - \gamma) - pk_2(1 - \gamma) + 0.5k_2L(1 + \gamma) - \gamma = 0 \tag{20}$$

Thus k_c, τ_I and τ_D are calculated from the above equations using the tuning parameter α, β and γ.

3 Robustness analysis

It is essential to carry out the robustness study of any designed controller to take care of model uncertainties. The sensitivity function should be small at low frequency for a controller to provide a good load rejection. The sensitivity function can be defined by the following equation

$$S(s) = \frac{1}{1 + G(s)G_c(s)} \tag{21}$$

The complementary sensitivity function can be given as

$$T(s) = 1 - S(s) \tag{22}$$

$$T(s) = \frac{G(s)G_c(s)}{1 + G(s)G_c(s)} \tag{23}$$

The set point tracking information of a controller can be obtained from the complementary sensitivity function. The value of the complementary sensitivity function *T(s)* should be close to unity. The robustness performance of the manipulated variable or the controller output is measured by the maximum sensitivity function (*Ms*) which is defined as

$$Ms = \max_{\omega} |S(\omega j)| = \max_{\omega} \left| \frac{1}{1 + G(j\omega)G_c(j\omega)} \right| \tag{24}$$

It is recommended to get a small value of *Ms* value for a good robust controller

The values of α, β and γ are chosen to stabilize the system and provide a better performance. The guideline for determining α, β and γ values are studied especially for the case of NMP SOPTD model. The optimal tuning parameters α, β and γ are estimated by minimizing the Integral Time weighted Absolute Error (ITAE) for the servo problem using the MATLAB *fminsearch* solver aimed at providing lower maximum sensitivity function and maintaining stability control. As non-minimum phase systems are highly sensitive to change in controller settings and tuning parameters, it was observed by simulations that for the following case studies, variance of α and the ratios α/β, α/γ gave better performance and achieved robustness (lower *Ms* value for obtained controller settings) and also stabilized the system. Hence, the selection of tuning parameters was a trade-off for all three requirements.

4 Simulation studies

Three examples of non-minimum second order stable system are considered for evaluation of proposed method. A rigorous simulation studies have been performed to get the optimal tuning parameter. To obtain the optimum PID settings, the initial PID settings obtained must be adjusted repeatedly using the tuning parameters such as α, β and γ through computer simulations until the lower Ms index and the required closed response are achieved. The efficacy of the controller performance is measured in terms of time integral errors, total variation in manipulated variable and settling time. The closed loop servo and regulatory response are obtained for all case studies and compared with the reported methods and quantifying in terms of time integral errors such as Integral of Absolute Error (IAE), Integral of Time weighted Absolute Error (ITAE) and Integral of Square Error (ISE). The smoothness of functioning of the controller is given in terms of Total Variation (TV). TV can be defined as $TV = \sum_{i=0}^{N} u_{i+1} - u_i$where N is the number of data points over the response and u is the manipulated variable.

Case study-1 (Fermenter)

The transient nonlinear mathematical model of a Fermenter in which the microbial growth is assumed to follow Monod kinetics is given by the following equation [25, 26].

$$\frac{dX}{dt} = (\mu - K_d)X - Xu \tag{25}$$

$$\frac{dS}{dt} = (\frac{\mu}{Y} - m)X - (S - S_t)u \tag{26}$$

$$\mu = \frac{\mu_{\max} S}{K_m + S} \tag{27}$$

In the above model equations, X is the cell concentration, μ the specific growth rate, m the maintenance, K_d the decay rate, Y the yield coefficient, u the dilution rate, K_m the Monod constant and S the substrate concentration. The Fermenter model (eqs. (25), (26) and (27)) is linearized around the stable operating point $X = 0.3018$ g/l and $S = 0.0452$ g/l with values $\mu_{max} = 0.4$/h, m = 0, $K_d = 0.04$/h, $K_m = 0.05$ g/l, $Y = 0.4$ g/g, $u = 0.15$/h. The obtained linear transfer function model is given by the following

$$\frac{\mathrm{X(s)}}{\mathrm{u(s)}} = \frac{0.2803(1 - 3.4063s)}{3.1638s^2 + 5.7382s + 1} e^{-0.3s} \tag{28}$$

For the proposed method, controller settings obtained are $k_c = 3.1944$, $\tau_I = 5.7956$, $\tau_D = 0.4135$ by considering $\alpha = 0.1514$, $\beta = -5.475\alpha$, $\gamma = 1.925\alpha$ values respectively. The PID settings by IMC method are $k_c = 2.8019$, $\tau_I = 5.7382$ and $\tau_D = 0.5514$ and designed by the stability analysis method are $k_c = 3.1944$, $\tau_I = 5.7382$, and $\tau_D = 0.5514$. The performance is evaluated in the closed loop system for a unit step change in the set point and unit change in load as shown in Figure 1 and Figure 2 respectively. From Figure 1, it can be seen that the closed loop characteristics such as rise time, settling time are less when compared to the existing methods listed in Table 1. It can also be observed from both the closed loop responses that the proposed method achieves a smoother undershoot than the other two methods. The quantification of the controller performance has been carried out in terms of the time integral errors and enlisted in Table 1. It can be observed from Table 1 that the integral absolute error and the integral time weighted absolute error are less for the proposed method for both servo and regulatory problems. The proposed method shows significant improvement in the controller performance by reducing the ITAE index by 61 % compared to the IMC method. The control actions are given in Figure 3 and Figure 4 for both servo and regulatory problem respectively. It can be seen from Table 1 (TV index) that the proposed method shows smoother control action for both servo and regulatory problem It is observed from Table 1 that the TV value for the servo response improved by 51 % and the TV index for regulatory response improved by 97 %. Kharitonov's theorem is used to determine the regions of stability for the model parameters. It can be observed from the Table 2 that the proposed controller can stabilize the system for ±253 % of perturbations in time delay. In the presence of uncertainty in process parameters, the proposed controller provides the best performance and can stabilize the system for a wider range of variations in model parameters as given in Table 2.

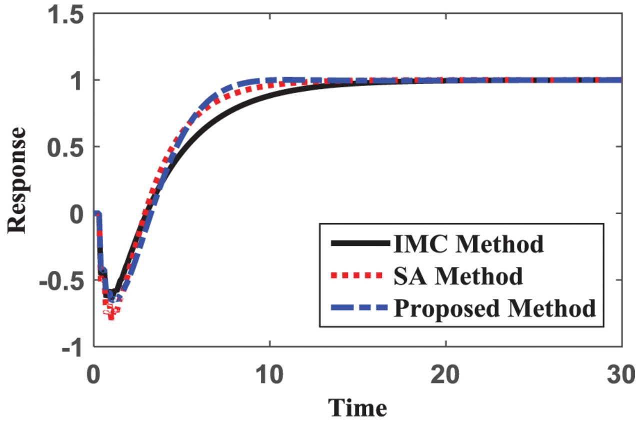

Figure 1: Closed loop Servo performance of case study-1using the PID settings obtained from the IMC method, the Stability Analysis (SA) method and the proposed method.

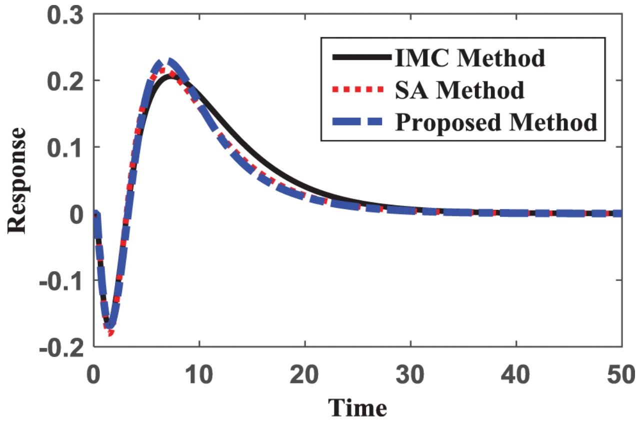

Figure 2: Regulatory performance of case study-1using the PID settings obtained from IMC method, SA method and proposed method.

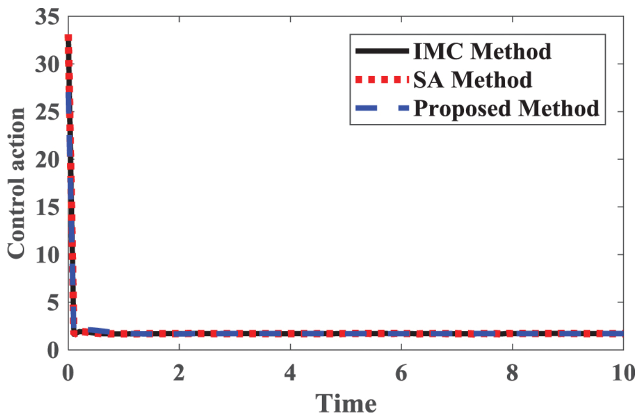

Figure 3: Control action obtained from Servo response using controller settings of IMC method, SA method and proposed method. (case-1).

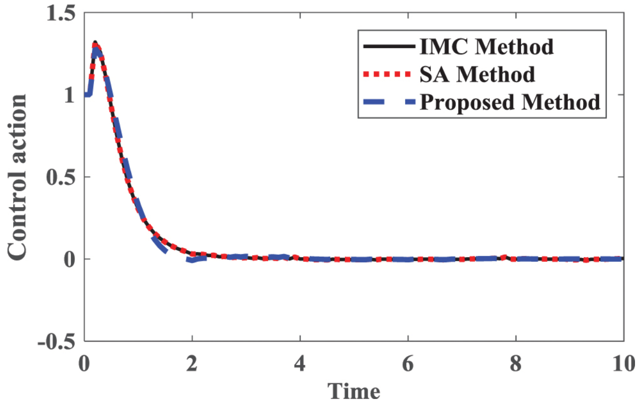

Figure 4: Control action obtained from Regulatory response using controller settings of IMC method, SA method and proposed method. (case-1).

Table 1: Error and close loop characteristics of all case studies.

Case studies	Design Method	Servo Problem		Regulatory Problem		TV		Rise Time(s)	Settling Time(s)
		IAE	ITAE	IAE	ITAE	Servo	Regulatory		
	IMC	7.328	26.277	2.631	27.608	163.71	3.7145	1.01	2.16
Case 1	SA	6.427	17.373	2.418	22.808	197.94	5.9133	1.01	2.16
	Present	**6.372**	**16.306**	**2.398**	**22.384**	**131.23**	**2.987**	**0.802**	**1.63**
	IMC	1.036	0.554	0.658	1.121	32.698	1.816	7.17	14.1
Case 2	SA	1.036	0.554	0.658	1.121	32.698	1.816	4.86	10.3
	Present	**0.911**	**0.533**	**0.651**	**1.103**	**26.654**	**1.748**	**3.58**	**8.24**
	IMC	36.880	290.34	**3.448**	57.035	329.02	11.010	14.7	30.4
Case 3	SA	36.819	287.66	3.455	**56.907**	337.163	11.9678	14.5	30.1
	Present	**36.423**	**267.67**	3.575	57.261	**148.81**	**6.347**	**11.3**	**26.7**

[1] SA: Stability Analysis Method

Table 2: Stability regions for model parameters (Kharitonov's Theorem).

Case studies	Method	%L	$\%k_p$	$\%a_1$	$\%a_2$
	IMC	±88	± 98	±60	±44
Case study 1	Proposed	±253	± 99	±95	±47
	IMC	±73	±98	±48	±22
Case study 2	Proposed	±390	±99	±99	±62
	IMC	±76	±30	±77	±22
Case study 3	Proposed	±155	±28	±171	±21

Case study 2 (Isothermal CSTR)

The following non-linear Van De Vusse isothermal CSTR [13] is considered here. The product B is desired in following reaction sequence [27, 28]

$$A \xrightarrow{k1} 3B \xrightarrow{k2} C \tag{29}$$

$$2A \xrightarrow{k3} D \tag{30}$$

The transient equations for the reactor are given as

$$\frac{\mathrm{d}C_A}{dt} = \frac{F}{V}(C_{A,f} - C_A) - K_1 C_A - K_3 C_A^2 \tag{31}$$

$$\frac{dC_B}{dt} = -\frac{F}{V}C_B + K_1C_A - K_2C_B \tag{32}$$

In the above modelling equations, F is feed flow rate, K_1, K_2, K_3 are the reaction rate constants, C_A the concentration of A in the reactor, C_{Af} the feed concentration of A, C_B the concentration of B in reactor and V the volume of the reactor. The parameters of the CSTR are given as $F/V = 0.5714/\text{min}$, $K_1 = 0.833/\text{min}$, $K_2 = 1.66/\text{min}$, $K_3 = 0.166/\text{min}$, $C_{Af} = 10$ gmol/l, $C_A = 3$ gmol/L, $C_B = 1.1170$ gmol/L. The local linearized transfer function is given by [29]

$$\frac{X(s)}{u(s)} = \frac{-1.117s + 3.1472}{s^2 + 4.6429s + 5.3821}e^{-0.1s} \tag{33}$$

In this case study, the values of the tuning parameters obtained for the proportional integral derivative (PID) controller are kc = 1.4832,¨I = 0.8627,¨D = 0.1723, taking into account the tuning parameters as α = 0.1517, β = −1.009α, μ = 2.586α. The PID settings obtained by the IMC method are k_c = 1.4685, τ_I = 0.8627, τ_D = 0.2154 and are based on the stability analysis method: k_c = 1.4685, τ_I = 0.8627, τ_D = 0.2154. The performance of the closed loop is evaluated for a unit step change in the set point and unit change in the load as shown in Figure 5 and Figure 6 respectively. It is evident from Figure 5 and Figure 6 that the proposed method achieves superior closed loop servo and regulatory responses. Superior closed loop servo and regulatory responses are achieved by the proposed method. Table 1 shows the closed loop characteristics such as settling time and rise time, performance criteria such as integral of absolute error (IAE) and integral of time weighted absolute error (ITAE) for the proposed method, stability analysis method and IMC method. The minimum values of IAE and ITAE indices proves the stability and efficacy of the proposed method to control the NMP SOPTD system. The proposed controller gives a rise time of 3.58 seconds and a settling time of 8.24 seconds, which is quite low when compared to the other two methods. The variation of manipulated variable for the servo and regulatory system is evaluated and shown in Figure 7 and Figure 8. There is a 22 % of improvement in the TV value than that of the IMC and stability analysis method for the servo response. Table 2 shows the stability regions for the model parameters calculated using Kharitnov's Theorem. It can be observed from the Table 2 that the proposed controller can stabilize the system for ±390 % of perturbations in time delay. In the presence of uncertainty in process parameters, the proposed controller provides the best performance and can stabilize the system for a wider range of variations in model parameters.

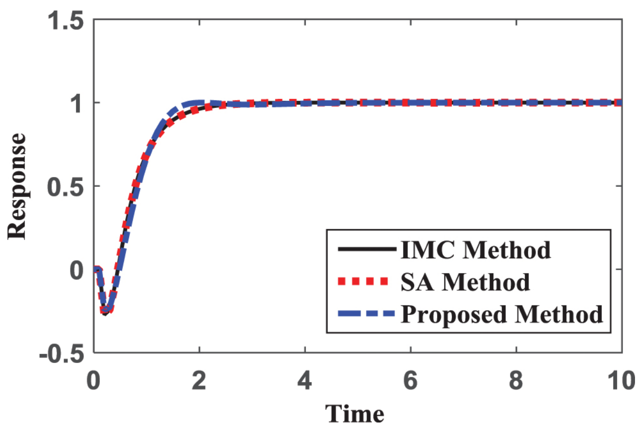

Figure 5: Closed loop Servo performance of case study-2using the PID settings obtained from IMC method, SA method and proposed method.

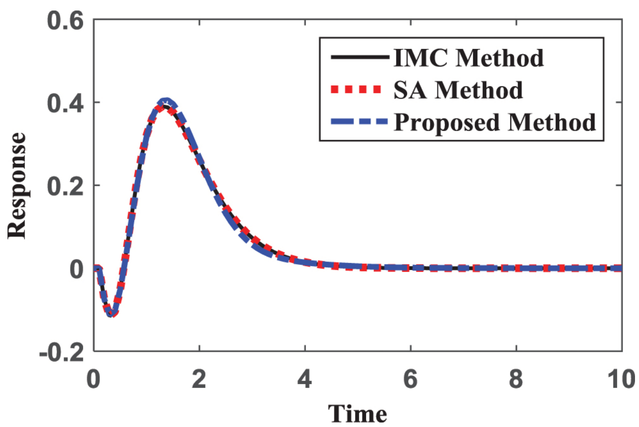

Figure 6: Regulatory performance of case study-2 using the PID settings obtained from IMC method, SA method and proposed method.

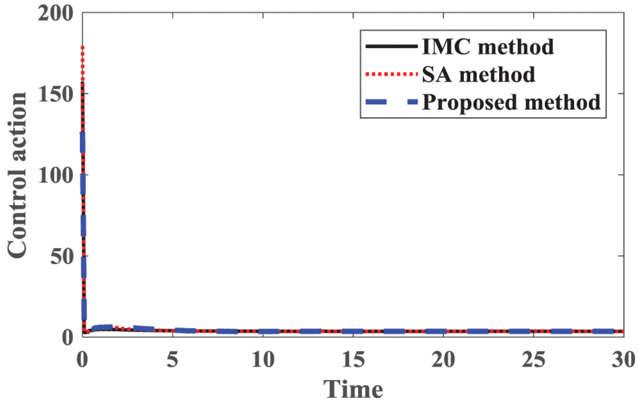

Figure 7: Control action obtained from Servo response using controller settings of IMC method, SA method and proposed method. (case-2).

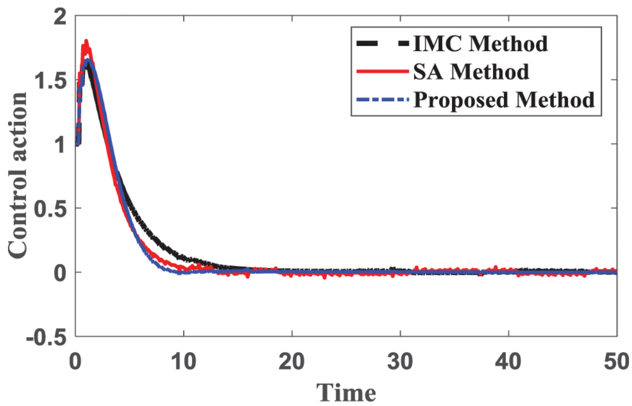

Figure 8: Control action obtained from Regulatory response using controller settings of IMC method, SA method and proposed method. (case-2).

Case study-3 (Fermenter)

On linearization of equations (eqs. (19–21)) at stable operating point $X = 0.379$ g/l and $S = 0.019$ g/l with values $\mu = 0.4$/h, $m = 0.01$/h, $K_d = 0$, $K_m = 0.05$ g/l, $Y = 0.4$ g/g, $u = 0.11$/h, the following transfer function relating cell concentration and dilution rate can be obtained [30, 31].

$$\frac{\mathrm{X(s)}}{\mathrm{u(s)}} = \frac{0.0291(1 - 28.7121s)}{2.2024s^2 + 9.0155s + 1} e^{-0.3s} \tag{34}$$

The controller settings for the present method is found to be $k_c = 8.4699$, $\tau_I = 9.0155$ and $\tau_D = 0.1221$ by considering $\alpha = -0.0362$, $\beta = 87.78\alpha$ and $\gamma = 0.0362\alpha$. The PID settings for the IMC method are $k_c = 8.3705$, $\tau_I = 9.0155$ and $\tau_D = 0.2443$ and designed by stability analysis method are $k_c = 8.386$, $\tau_I = 9.0155$ and $\tau_D = 0.2443$. These set of PID settings are evaluated on the Fermenter and obtained the servo and regulatory responses by introducing a unit step change in set point and load. The obtained are shown in Figure 9 and Figure 10. Quantification of the closed loop performances are carried out in terms of time integral errors. Table 1 shows the comparison of controller parameters such as settling time and rise time, performance criterion such as integral of absolute error (IAE) and integral of time weighted absolute error (ITAE) for the proposed method, Stability analysis method and IMC method. The smoother control action is achieved for both servo and regulatory (Figure 11 and Figure 12) problems. Compared to the IMC method, the TV value for the servo response improved by 148 % and the TV values for the regulatory response improved by 73 %. Compared to the stability analysis method, the TV value for the servo response improved by 126 % and the TV values for the regulatory response improved by 88 %. From the proposed controller, a settling time and rise time of 26.7 seconds and 11.3 seconds respectively are observed which are comparably less than the stability analysis and the IMC controller. Kharitonov's theorem is used to determine the regions of stability for the model parameters. It can be observed from the Table 2 that the proposed controller can stabilize the system for ±155 % of perturbations in time delay. In the presence of uncertainty in process parameters, it is shown that the proposed controller delivers the best performance and is able to stabilize the system for a wider range of system parameter variations.

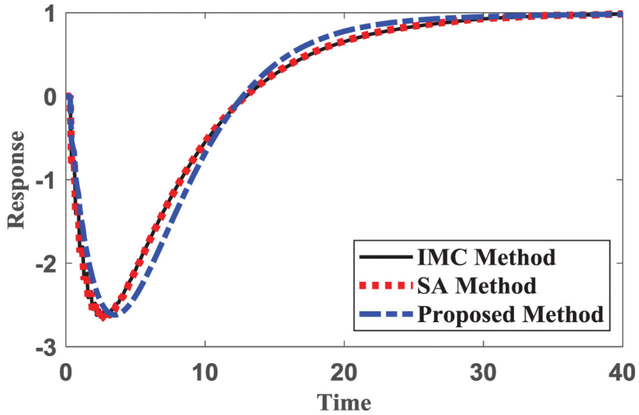

Figure 9: Closed loop Servo performance of case study-3using the PID settings obtained from IMC method, SA method and proposed method.

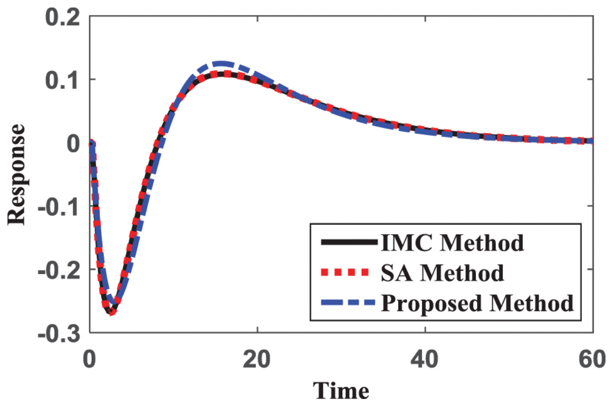

Figure 10: Regulatory performance of case study-2 using the PID settings obtained from IMC method, SA method and proposed method.

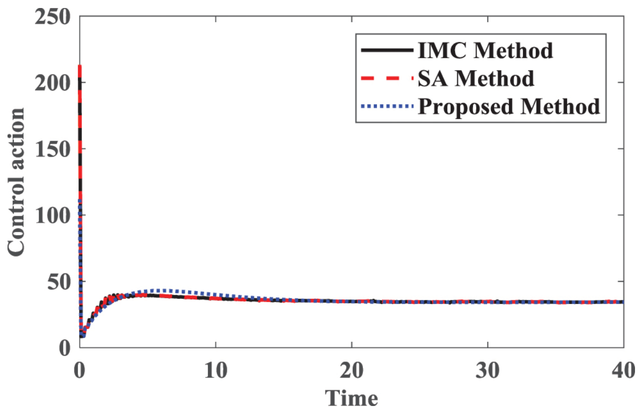

Figure 11: Control action obtained from Servo response using controller settings of IMC method, SA method and proposed method. (case-2).

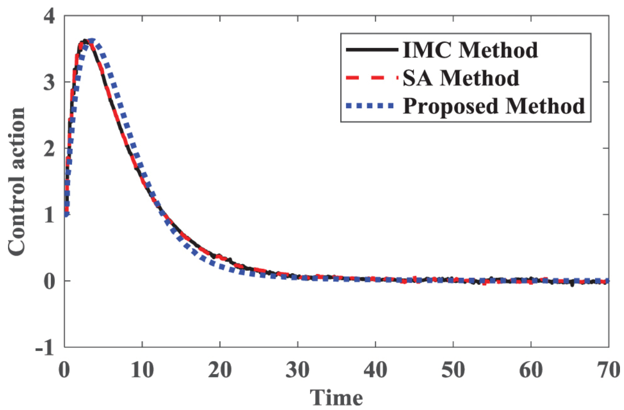

Figure 12: Control action obtained from Regulatory response using controller settings of IMC method, SA method and proposed method. (case-2).

5 Conclusions

A tuning method is proposed to design a controller for the non-minimum phase second order plus time delay systems. Unlike the other methods, the method used in this study is very simple and the tuning techniques are easy. The proposed method is compared with those methods reported in literature such as IMC and stability analysis method. From the simulation studies, it is shown that a significant enhancement in the closed loop performance has been observed in all the case studies compared to the existing methods reported in literature. The proposed method shows better closed loop response in terms of integral absolute error and the integral time weighted absolute error. The smoother control action is achieved by the proposed method for both servo and regulatory problems. It is also observed that the proposed method is capable of stabilizing the system for a larger range of system parameter variations. The performance improvement of the proposed method over IMC method and stability analysis method have shown that the proposed controller can be able to control the non-minimum phase SOPTD systems.

Appendix

A Kharitonov's Theorem [28]

This theorem gives interval for the coefficient of the characteristic equation for which the system is stable by determining the stability of vortex polynomial obtained for the boundary range of the coefficients.

The closed loop characteristic equation is given by

$$G(s) = 1 + GG_c = 0 \tag{35}$$

The characteristic equation obtained by applying Taylor's series expansion for time delay approximation for the interval system is stated as follows

$$G(s) = a_0 + a_1 s + a_2 s^2 + a_3 s^3 + \ldots\ldots + a_n s^n \tag{36}$$

Where, $a_i \in [a_{il}, a_{iu}]$,

For i = 1,2, … n., a_{il} is the lower limit and a_{iu} is the upper limit,

The characteristic polynomial is said to be stable only if all four khairtonov polynomial are stable. Their stability is found by applying Routh hurwitz criterion to each equation. The khairtonov's polynomials are gives as

$$G1(s) = a_{0l} + a_{1l} s + a_{2u} s^2 + a_{3u} s^3 + \ldots\ldots \tag{37}$$

$$G2(s) = a_{0l} + a_{1u} s + a_{2u} s^2 + a_{3l} s^3 + \ldots\ldots \tag{38}$$

$$G3(s) = a_{0u} + a_{1l} s + a_{2l} s^2 + a_{3u} s^3 + \ldots\ldots \tag{39}$$

$$G4(s) = a_{0u} + a_{1u} s + a_{2l} s^2 + a_{3l} s^3 + \ldots\ldots \tag{40}$$

The coefficient polynomial (36) is stable if and only if all the four vertex polynomials (37–38) are stable. Initial values of k_p, a_1 and a_2 are fixed and perturbation in time delay L is substituted with limits $(L - \Delta L) < L < (L + \Delta L)$ in coefficients and Kharitonov's polynomials are obtained. These polynomial's stability is checked with Routh–Hurwitz method. In similar procedure, stability regions for k_p, a_1 and a_2 are obtained by varying objective parameter and keeping other parameters constant.

Nomenclature

Greek alphabets:

α Coefficient for s^3

β Coefficient for s^2

γ Coefficient for s

τ Process time constant

τ_I Integral time constant

τ_D Derivative time constant

μ Growth rate for Monod kinetics

Abbreviations:

PID Proportional, Integral, Derivative controller

NMP Non-Minimum Phase

FOPTD First Order plus Time Delay

SOPTD Second Order plus Time Delay

USOPTD Unstable second order plus time delay

IMC Internal Model Control

SA Stability Analysis

IAE Integral of Absolute Error

ITAE Integral of Time weighted Absolute Error

ISE Integral of Squared Error

TV Total Variation in manipulated variable

G Process transfer function

G_C Controller transfer function

MS Maximum Sensitivity function

SOPTDZ Second Order Plus Time Delay with Zeros

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