Self-Similar Behavior of Highway Road Traffic and Performance Analysis at Toll Plazas

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Abstract: Until recently, the Poisson process has been used to model internet and road traffic queues. It has been established that internet traffic exhibits self-similarity, which is very different from the Poisson process. Motivated by this, efforts have been made to examine whether road traffic is also self-similar. Earlier efforts in this direction indicate that road traffic is indeed self-similar. To substantiate this, this paper examines, by various methods, whether real time traffic on a busy national highway in India is self-similar. The results from this examination prove that the traffic observed on the highway is self-similar. This paper also presents a novel method based on percentiles for computing the Hurst parameter, which is an indicator for the intensity of self-similarity. The paper also validates the percentile method with two other existing methods. Additionally, the traffic at a toll plaza on the highway has been modelled as queueing system, and performance measures have also been computed, namely, mean queue length and busy period distribution. The numerical results clearly demonstrate that the analysis presented in this paper can be useful for improved designs of toll plazas. **DOI:** 10.1061/(ASCE)TE.1943-5436.0000427. *© 2012 American Society of Civil Engineers.*

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Introduction

Vehicle arrival pattern is one of the fundamental concerns in the planning and design of highways. Until recently, the Poisson process has been used to model road traffic, irrespective of traffic intensity. This is similar to the practice in cases of Ethernet, LAN, WAN, and WWW traffic. However, seminal studies (Leland et al. 1994; Crovella and Bestavros 1997) have revealed that IP packet traffic in said networks tends to be bursty in many time scales. This bursty traffic can be characterized mathematically as self-similar or long-range dependence (LRD). The work of Paxson and Floyd (1995) shows that Poisson process can not emulate the self-similar network traffic. The Markovian arrival process (MAP) emulating self-similar traffic is fitted over desired time scales by equating second-order statistics of the counts (Anderson and Nielsen 1998; Yoshihara et al. 2001; Shao et al. 2005). Consequently, a question has been raised among researchers of transportation engineering: does the Poisson process emulate highway vehicle traffic, particularly when the traffic intensity is very high? Through a simulation study, (Nagatani 2005) concluded that a single vehicle passing through a sequence of traffic lights exhibits self-similar behavior. This study opened the problem of the existence of self-similarity in road traffic. Meng and Khoo (2009) investigated real-time road traffic and concluded that traffic is self-similar. Therefore, such traffic cannot be emulated by the Poisson process. If the arrival pattern of vehicle is Poisson at a particular point, for example at toll plaza, then one can readily compute various metrics such as mean waiting time and busy period distribution by using queueing theory techniques. Computation of these metrics therefore needs to be investigated, when the road traffic is self-similar. Very little focus has been made on the self-similar characteristics of the arrival paterns of vehicles on road traffic and their consequences. Hence, this has been examined in this paper.

The objective of this paper, therefore, is two-fold. First, the study examines whether vehicle arrival pattern on highways is self-similar or not. This is to supplement earlier results using real-time data. Second, certain performance metrics are computed by modeling the traffic at a toll plaza as a queueing system with self-similar input traffic for deriving useful inferences. This kind of analysis is useful in the continuous improvement of highways where the traffic is self-similar, and can enable optimal, improved planning and design of toll plazas.

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Self-Similarity and Long-Range Dependence

The definition of exact second-order self-similar process is described in the following. Arrival instants are modeled as point process. Divide the time axis into disjoint intervals of unit length and let $X = \{X_t{:}t = 1, 2, \ldots.\}$ be the number of points (arrivals) in the tth interval. Let X be a second-order stationary process with variance σ^2 and the autocorrelation function $\gamma(k)$, $k \geq 0$ is given by

$$\gamma(k) = \frac{\mathrm{Cov}(X_t, X_{t+k})}{\mathrm{Var}(X_t)} \tag{1}$$

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For each $m = 1, 2, 3\ldots$, let a new time-series $X^{(m)} = \{X_t^{(m)}/t = 1, 2, 3\ldots\}$ be obtained averaging the original time-series X over non-overlapping blocks of size m:

$$X_t^{(m)} = \frac{1}{m}\sum_{i=1}^{m} X_{(t-1)m+i}, \qquad t = 1, 2\ldots. \tag{2}$$

This new series $X^{(m)}$, for each m, is also a second-order stationary process with autocorrelation function $\gamma^m(k)$.

Definition 1: The process 'X' is said to be exactly second-order self-similar with Hurst parameter $H = 1 - \frac{\beta}{2}$ and variance σ^2 if

$$\gamma(k) = \frac{\sigma^2}{2}[(k+1)^{2H} - 2k^{2H} + (k-1)^{2H}], \quad \forall\ k \geq 1. \tag{3}$$

Definition 2: The process 'X' is said to be asymptotically second-order self-similar with Hurst parameter $H = 1 - \frac{\beta}{2}$ and variance σ^2 if

$$\underset{m\to\infty}{Lim} \times \gamma^m(k) = \frac{\sigma^2}{2}[(k+1)^{2H} - 2k^{2H} + (k-1)^{2H}], \quad \forall\ k \geq 1. \tag{4}$$

In terms of variance of the averaged process, a similar process can be definded as follows.

Definition 3: The process 'X' is said to be exactly second-order self-similar with Hurst parameter $H = 1 - \frac{\beta}{2}$ and variance σ^2 if

$$Var(X^{(m)}) = \sigma^2 m^{-\beta}, \quad \forall\ m \geq 1. \tag{5}$$

LRD and short-range dependence (SRD) can be differentiated as follows: for $H \neq 0.5$, Eq. (3) shows that $\gamma(k) = H(2H-1)k^{2H-2}$ as $k \to \infty$, and thus:

$$\sum_k \gamma(k) \sim c\sum_k k^{-\beta}, \qquad c = H(2H-1). \tag{6}$$

The series $c\sum_k k^{-\beta}$ is divergent if $0.5 < H < 1$ or $0 < \beta < 1$, otherwise they are convergent, because they are a positive term series. Accordingly, the left-hand series $\sum_k \gamma(k)$ is divergent if $0.5 < H < 1$ or $0 < \beta < 1$, otherwise they are convergent. Hence, for $0.5 < H < 1$, the autocorrelation function decays slowly, that is, hyperbolically; in this case, the process X is called LRD. The process X is called SRD if $0 < H < 0.5$ or the autocorrelation function is summable (finite).

Data Description

As discussed previously, this paper pimarily focuses on vehicle arrival patterns on busy highways. For this, real-time data were studied, provided by V R TECHNICHE Consultants Pvt. Ltd., India. For ready reference, these data are given in Tables 1–7. The data used for simulations in this study were collected from toll traffic reports at one of the three operating toll plazas on the Delhi-Gurgaon section of National Highway 8 (NH8) in India. The Delhi-Gurgaon section of NH8 is a 6/8-lane build, operate, and transfer (BOT) toll road with a 20-year concession period. The data were accumulated during the period June14 to June 20, 2010. Gurgaon is one of the fastest-growing cities in the national capital region of India, and the Delhi-Gurgaon section of NH8 is one of the busiest highway sections in India. The Delhi-Gurgaon section has three toll plazas. Out of these, the toll plaza at km 24 (between Delhi and Gurgaon) is busiest and has 32 toll lanes, as shown in Fig. 1. The km 24 toll plaza with 32 toll lanes is the largest toll plaza in India and is one of the largest in the world.

Table 1. Collection of Data on Day 1

	June 14, 2010					
Hour	Car	Light commercial vehicles (LCV)	Mini bus	Truck	Bus	Multi-axle vehicles (MAV)
00–01	1,782	178	9	237	48	615
01–02	988	219	4	263	40	601
02–03	703	215	4	258	35	532
03–04	654	213	9	197	44	417
04–05	1,164	162	8	113	71	192
05–06	2,049	188	14	65	103	172
06–07	2,918	149	15	40	121	75
07–08	6,561	89	53	16	194	28
08–09	12,910	43	96	3	309	14
09–10	11,500	36	42	1	163	17
10–11	10,169	118	18	12	144	25
11–12	7,796	158	13	16	140	45
12–13	6,636	207	14	16	131	50
13–14	6,478	215	13	21	110	45
14–15	6,806	239	15	19	125	55
15–16	6,919	239	15	12	124	34
16–17	8,232	172	28	7	134	23
17–18	10,801	48	55	1	192	19
18–19	12,091	49	41	2	177	20
19–20	11,341	72	19	6	138	27
20–21	8,396	174	15	65	99	138
21–22	4,956	302	13	98	82	195
22–23	3,896	220	14	188	63	435
23–24	2,859	201	6	170	66	527
Total	148,605	3,906	533	1,826	2,853	4,301

Table 2. Collection of Data on Day 2

	June 15, 2010					
Hour	Car	Light commercial vehicles (LCV)	Mini bus	Truck	Bus	Multi-axle vehicles (MAV)
00–01	1,667	231	15	249	61	656
01–02	1,024	301	7	320	37	717
02–03	916	277	3	325	33	661
03–04	952	289	2	226	45	503
04–05	1,282	220	4	161	62	263
05–06	2,195	251	11	97	73	189
06–07	2,821	197	14	58	129	91
07–08	6,799	123	67	17	205	35
08–09	13,251	40	107	3	299	25
09–10	11,793	38	40	4	166	11
10–11	8,908	111	13	5	143	24
11–12	7,272	207	13	14	132	31
12–13	7,075	241	15	23	122	70
13–14	6,233	230	10	24	102	42
14–15	6,671	248	9	20	140	47
15–16	6,818	223	21	19	119	41
16–17	8,382	177	34	6	132	23
17–18	10,335	69	55	1	192	20
18–19	11,683	60	29	4	176	18
19–20	11,148	65	28	5	124	26
20–21	8,191	182	14	54	104	76
21–22	5,667	304	10	114	91	257
22–23	4,111	342	12	169	79	443
23–24	2,945	322	12	197	75	559
Total	148,139	4,748	545	2,115	2,841	4,828

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Table 3. Collection of Data on Day 3

	June 16, 2010					
Hour	Car	Light commercial vehicles (LCV)	Mini bus	Truck	Bus	Multi-axle vehicles (MAV)
00–01	1,941	277	14	277	57	652
01–02	1,080	275	9	270	38	702
02–03	950	305	6	300	45	736
03–04	866	333	7	235	45	459
04–05	1,328	273	5	147	67	306
05–06	2,075	255	6	109	79	191
06–07	2,678	207	18	63	123	129
07–08	6,422	145	63	30	215	59
08–09	12,864	43	117	14	303	21
09–10	12,067	53	39	4	147	21
10–11	9,446	130	15	14	143	35
11–12	7,736	215	5	17	115	44
12–13	6,589	258	16	14	139	55
13–14	6,861	204	13	18	109	37
14–15	6,956	260	18	21	125	46
15–16	7,042	246	19	18	128	41
16–17	8,217	181	22	5	123	28
17–18	10,752	71	47	2	200	19
18–19	11,711	56	29	2	166	25
19–20	11,042	89	21	8	143	32
20–21	8,323	205	14	75	99	138
21–22	5,864	283	11	121	108	235
22–23	4,536	353	16	187	91	541
23–24	2,875	276	8	163	61	517
Total	150,221	4,993	538	2,114	2,869	5,069

Table 4. Collection of Data on Day 4

	June 17, 2010					
Hour	Car	Light commercial vehicles (LCV)	Mini bus	Truck	Bus	Multi-axle vehicles (MAV)
00–01	2,022	268	12	220	79	593
01–02	1,168	279	8	286	35	640
02–03	1,015	290	5	286	25	604
03–04	889	354	6	226	45	435
04–05	1,334	251	7	142	58	282
05–06	2,188	255	14	100	88	197
06–07	2,828	188	22	50	112	87
07–08	6,648	130	58	31	213	44
08–09	12,922	46	120	3	302	17
09–10	11,843	46	34	2	147	8
10–11	9,367	135	12	15	141	27
11–12	7,583	219	9	14	144	38
12–13	6,665	236	10	16	121	52
13–14	6,456	245	14	22	104	33
14–15	6,779	243	5	25	128	30
15–16	7,020	264	19	20	138	24
16–17	8,429	186	26	12	130	19
17–18	10,811	57	55	2	199	12
18–19	11,797	66	30	5	171	21
19–20	10,976	68	26	8	162	26
20–21	8,434	191	13	84	94	111
21–22	5,493	309	9	89	95	225
22–23	4,953	357	9	185	83	497
23–24	3,224	308	5	217	78	577
Total	150,844	4,991	528	2,060	2,892	4,599

Table 5. Collection of Data on Day 5

	June 18, 2010					
Hour	Car	Light commercial vehicles (LCV)	Mini bus	Truck	Bus	Multi-axle vehicles (MAV)
00–01	1,917	275	13	225	70	537
01–02	1,159	280	8	257	48	623
02–03	1,035	287	5	304	37	637
03–04	941	337	1	264	46	549
04–05	1,372	249	8	144	72	257
05–06	2,139	270	10	85	97	160
06–07	2,816	207	12	62	115	103
07–08	6,495	136	60	22	207	50
08–09	13,387	55	115	15	304	18
09–10	11,031	48	31	4	144	18
10–11	9,409	145	14	12	145	19
11–12	7,902	211	14	24	144	50
12–13	6,823	215	13	25	119	47
13–14	6,743	248	17	27	118	42
14–15	7,116	252	12	20	115	45
15–16	7,223	252	15	12	125	48
16–17	8,672	199	26	6	128	35
17–18	11,163	58	49	4	209	18
18–19	12,162	58	30	4	168	17
19–20	11,372	76	30	1	133	23
20–21	8,810	170	17	64	99	101
21–22	6,708	340	14	111	100	267
22–23	4,533	314	9	178	80	471
23–24	3,184	286	7	181	62	501
Total	154,112	4,968	530	2,051	2,885	4,636

Table 6. Collection of Data on Day 6

	June 19, 2010					
Hour	Car	Light commercial vehicles (LCV)	Mini bus	Truck	Bus	Multi-axle vehicles (MAV)
00–01	2,479	299	14	229	63	577
01–02	1,653	269	8	296	40	677
02–03	1,247	322	6	311	29	661
03–04	1,238	376	6	272	47	522
04–05	1,569	273	7	191	57	380
05–06	2,200	273	9	119	65	192
06–07	2,379	213	12	87	146	112
07–08	3,549	124	28	14	165	31
08–09	6,550	44	47	3	194	17
09–10	6,946	43	24	14	145	15
10–11	7,160	130	4	9	151	12
11–12	6,319	210	9	15	105	47
12–13	6,417	291	13	18	112	51
13–14	6,383	239	19	20	109	41
14–15	6,459	278	18	21	111	55
15–16	6,684	238	17	17	125	44
16–17	6,989	184	17	7	135	30
17–18	7,643	46	16	0	145	32
18–19	7,956	62	18	7	142	22
19–20	7,066	68	15	4	125	16
20–21	6,037	202	12	67	99	91
21–22	4,841	337	10	102	62	268
22–23	5,473	378	23	186	87	412
23–24	3,726	322	9	192	77	514
Total	118,963	5,221	361	2,201	2,536	4,819

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Table 7. Collection of Data on Day 7

	June 20, 2010					
Hour	Car	Light commercial vehicles (LCV)	Mini bus	Truck	Bus	Multi-axle vehicles (MAV)
00–01	2,194	240	10	216	53	521
01–02	1,551	255	9	254	22	621
02–03	1,043	269	4	245	44	600
03–04	926	297	5	239	42	511
04–05	1,308	249	7	198	79	303
05–06	1,777	203	8	119	85	227
06–07	1,934	163	9	43	113	99
07–08	2,358	114	14	22	155	42
08–09	3,158	104	10	4	127	18
09–10	3,665	87	8	2	134	11
10–11	4,336	113	8	14	142	45
11–12	5,237	145	2	20	105	64
12–13	5,321	142	6	23	106	50
13–14	5,050	143	7	21	97	47
14–15	4,903	153	9	25	116	49
15–16	5,145	124	9	17	133	47
16–17	5,625	106	9	7	121	29
17–18	5,855	83	9	2	125	24
18–19	6,147	97	11	4	122	17
19–20	5,729	83	10	6	102	23
20–21	5,312	143	4	71	92	93
21–22	4,887	221	17	125	67	250
22–23	4,117	303	9	164	76	459
23–24	3,287	295	7	170	85	509
Total	90,865	4,132	201	2,011	2,343	4,659

Fig. 1. km 24 toll plaza on the Delhi-Gurgaon section of NH8

All the toll plazas on Delhi-Gurgaon section of NH8 have electronic toll lanes with automatic vehicle detection and classification devises. The toll collection system at these toll plazas is fully automatic and manufactured by KAPSCH TraffiCom. As mentioned earlier, hourly traffic volume by classification was collected from toll collection reports. These data are obtained from the automatic vehicle detection and classification systems installed in each of the toll lanes at the toll plaza. The data were collected under four categories, namely, cars, light commercial vehicles (LCV) (minibuses and light goods vehicles), buses and standard trucks (i.e., trucks with 2-axles), and multi-axle vehicles (MAV) (goods vehicles with three or more axles).

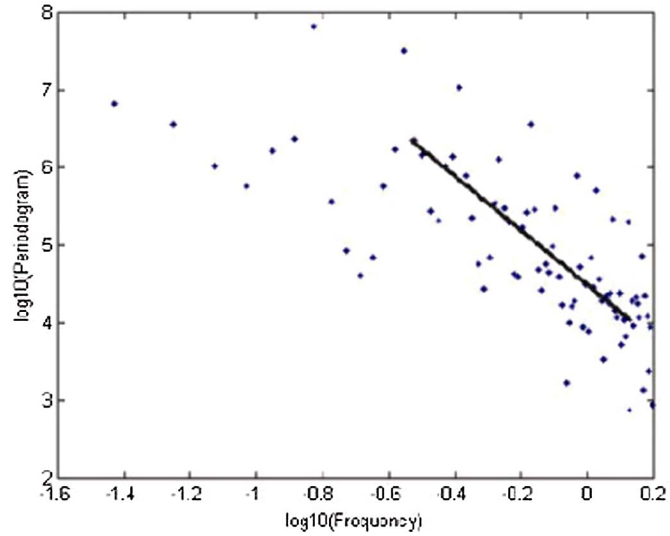

Fig. 2. log10 (peridogram) versus log10 (frequency)

Tests for Self-Similarity

The intensity of self-similarity is given by the Hurst parameter, H. The parameter H was named after the hydrologist H.E. Hurst, who spent many years investigating the problem of water storage and determining the level patterns of the Nile river. The parameter H has the range $0.5 \leq H \leq 1$. Estimation of H is a difficult task. Several methods are available to estimate the degree of self-similarity in a time-series. These include the periodogram method and the residuals of regression method. A new method, based on percentiles, has been proposed by Reddy (2011). For the first two methods, the free software SELFIS tool (2007) is employed to estimate the Hurst parameter, whereas the new method implemented in MATLAB (The Math Works, Inc., MA, USA) software.

Periodogram Method

In the frequency domain, the analysis of time-series is merely the analysis of a stationary process by means of its spectral representation. The periodogram is given by Sarkar (2007) as follows:

$$I_N(\lambda) = \frac{1}{2\pi N}\left|\sum_{j=0}^{N-1} X_j e^{j\lambda}\right|^2 \tag{7}$$

where λ = Fourier frequency; N = number of terms in the time-series; and X_j = data of the given series. To estimate H, first, one has to calculate this periodogram. Because $I_N(\lambda)$ is an estimator of the spectral density, a series with long-range dependence should have a periodogram, which is proportional to $|\lambda|^{1-2H}$ close to the origin. Then, a regression of the logarithm of the periodogram on the logarithm of the frequency λ should give a coefficient of $1-2H$. The slope of the fitted straight line is the estimate of $1-2H$. Using this method H value is computed for the data. The pertaining scattered data and trend line are depicted in Fig. 2. The obtained value of H in this case is 0.746.

Residuals of Regression Method

This method described by Peng et al. (1994) involves several steps. First, the series is divided into blocks of size m. Then, the partial sums of the series are calculated for each block, $Y(i), i = 1, 2, \ldots m$. A least square line is fitted to $Y(i)$ and the sample variance of the residuals is computed. This procedure is

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repeated for each of the blocks and the resulting sample variances are averaged. Thus, if the result is plotted on a log-log plot against m, a straight line with a slope of $2H$ should be the result; that is, residuals of variances against the number of blocks. Using this method, the H value is computed for the data given in the Appendix. The pertaining scattered data and trend line are depicted in Fig. 3. The obtained value of H in this case is 0.7546.

Percentile Method

In statistics, a percentile (or centile) is the value of a variable below which a certain percent of observations fall. There is no standard definition of percentile (David 2007). However, all definitions yield similar results when the number of observations is very large. One definition of percentile often given in texts is that the Pth percentile $P(1 \le P \le 100)$ of N ordered values is obtained by first calculating the rank

$$n = \frac{P \times N}{100} + \frac{1}{2} \tag{8}$$

Given a data set or time-series (t, Z_t), $t \ge 0$. First, the percentiles $(P_i,\ i = 1, 2, \ldots 100)$ are found for a given time-series using

$$P_i = \frac{i \times N}{100} + \frac{1}{2}; \qquad i = 1, 2, \ldots 100 \tag{9}$$

P_i (ith percentile) is a special type of average or partition values in descriptive statistics like quartiles (Q_1, Q_2, Q_3). The percentile number is plotted against percentiles on log scales. Then, a straight line or linear equation $Z_t = \beta t + c$ is obtained with the slope β. The Hurst parameter (H) is then computed by $H = 1 - \beta/2$. Using this method, the H value is computed for the data. The pertaining scattered data and trend line are depicted in Fig. 4. The obtained value of H in this case is 0.7675. A pertinent paper can be found on the site Web Hosting Talk.com (2008), which describes how the 95-percentile depends on the aggregation window size, and how this phenomenon justifies the mathematical definition of self-similarity. The advantages of this method are as follows: (1) data (however large it may be) is divided into 100 parts and the plotting involves only 100 points (percentile versus percentile number); (2) this method is matter of a simple empirical formula, unlike other methods.

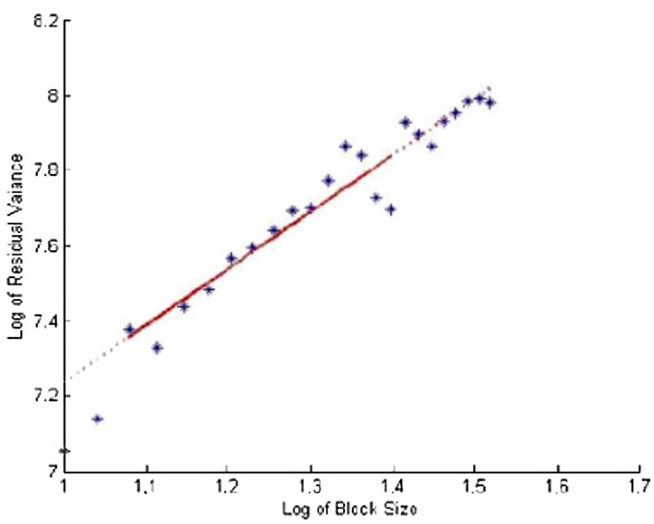

Fig. 3. log (residual of variance) versus log (block size)

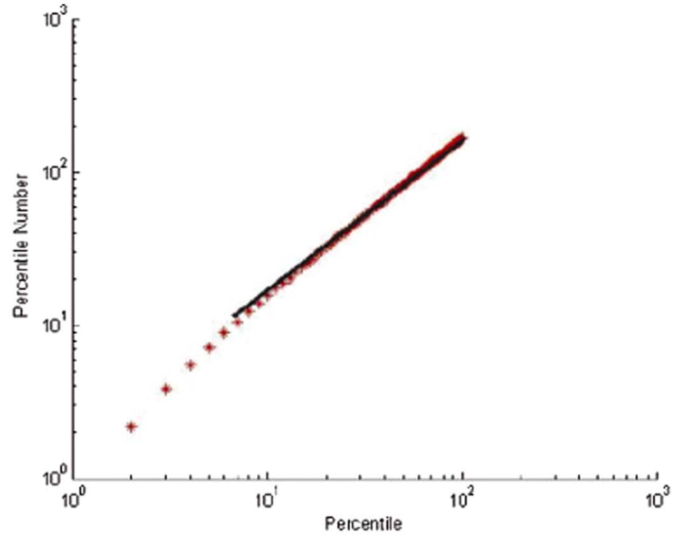

Fig. 4. log (percentile number) versus log (percentile)

Performance Metrics

The section presents numerical results of performance metrics; namely, mean queue length ($\bar{L}$) and busy period (B) distribution, by modeling the traffic at toll plaza as queueing system with self-similar input traffic. Mean queue length ($\bar{L}$) (Gunther 2000) of a queueing system is

$$\bar{L} = \frac{\rho^{0.5/(1-H)}}{(1-\rho)^{H/(1-H)}} \tag{10}$$

where ρ = traffic intensity. Mean queue length against traffic intensity is computed and the results are depicted in Fig. 5. As shown in Fig. 5, as ρ increases, the mean queue length increases, which is expected. Further more, as H increases, the mean queue length increases. This tendency agrees the authors' intuition. The busy period distribution for large queues is approximated by Ilkka Norris (Park and Willinger 2000) using large deviation techniques and is given by

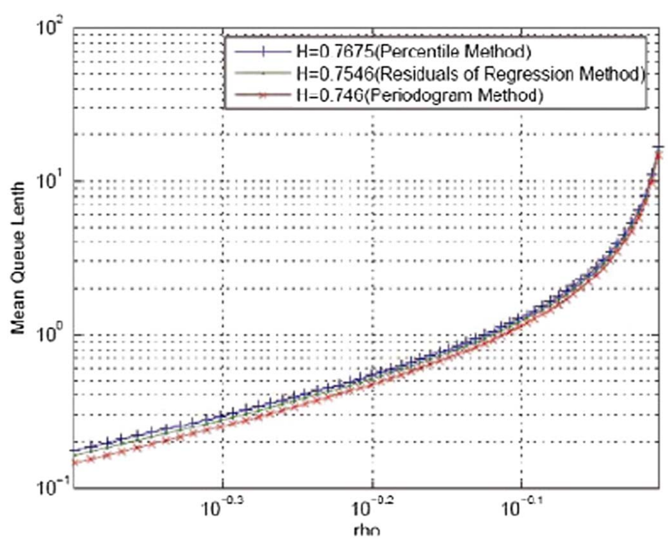

Fig. 5. Mean queue length versus traffic intensity

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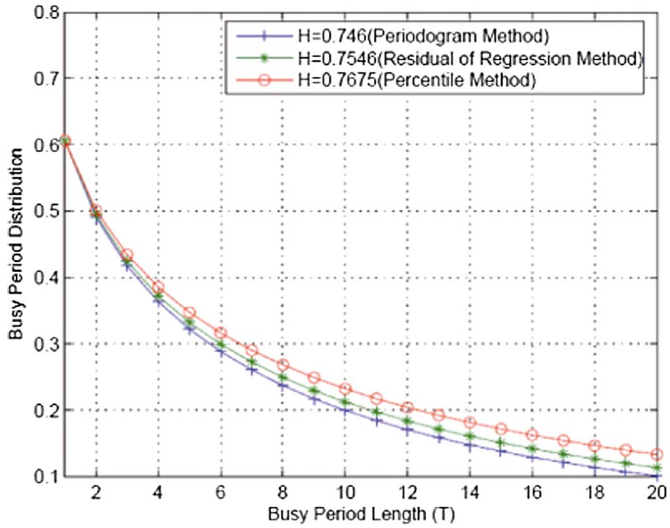

Fig. 6. Busy period distribution versus busy period length (T)

$$P(B > T) \approx e^{-T^{\frac{2-2H}{2}}} \tag{11}$$

In some books, this formula is even suggested for practical purposes. The busy period distribution is computed for three values of H and the results are depicted in Fig. 6. This figure clearly shows that when H is higher the busy period distribution will be higher.

Conclusions

In this paper, real-time highway traffic on a busy national highway has been proven to be self-similar. Data used for the study were provided by a leading consulting company in India. Various methods have been used to test the self-similarity. The obtained values of the Hurst parameter are reasonably close to each other. Mean queue length and busy period distribution are computed. Numerical results reveal that the mean queue length increases as ρ and H increase, and the busy period distribution is found to be higher if H is higher. The first metric, namely, mean queue length against traffic intensity, can be used to determine the optimal number of toll counters; whereas another metric, namely busy period length distribution, can be used to address the congestion problem. Based on this analysis, decisions can be generated for traffic diversion during the busy period. This kind of analysis in traffic engineering is useful for the improvement of designs of highways and toll plazas.

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